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BY

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"A few wrote in language which stirred the pulse of millions; others - no less important to the world - wrote in a prose which then, as now, fogs the brain."

R. L. Heilbroner,

The Worldly Philosophers.

ABSTRACT

This thesis is a literature study of the Harrod-Domar growth model. The Harrod and Domar models are first interpreted in the light of Harrod's and Domar's own words; after that the criticisms and refinements of their reviewers (Alexander, Baumol, Hamberg, Pilvin and Robinson) are scrutinized. In comparison, Samuelson's model is discussed, its similarities to and differences from the Harrod-Domar model are pointed out and Baumol's attempt at a reconciliation of at least one of the differences is shown. In another comparison Higgins' interpretation of Hansen's model is presented, after which Hicks' and Schumpeter's contributions to the theory of cycles in an expanding economy follow. It is seen that all the above economists had original ideas in regard to the problem of growth; that somehow however Harrod seems to give the most attractive allround theory. However, a more 'mathematical' presentation of his theory would have saved his reviewers much time, although the reviewers themselves have not always diminished the confusion by their own inexact treatment of the issues involved. The conclusion follows that strict logical reasoning is essential if economics is to become a more accessible science.

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INTRODUCTION

In the framework of history the following stages of growth can be distinguished: "the traditional society; the transitional society, in which the foundations of change are being laid; the society in the crucial process of 'take-off'; the maturing society, in which new methods and outlooks are spreading through the whole economy; and finally the society which has reached the age of high mass consumption."[1] Accompanying the first 'take-off' in history (in Great Britain between approximately 1783 and 1802 [2]) economics was established as a science with the publication of Adam Smith's Inquiry in 1776: the free market system, later to be called capitalism, was from now on under scrutiny. The take-off is that short stage of development, "in which the economy and the society of which it is part transform themselves so that economic growth becomes more or less automatic."[3] The early economists (especially Smith, Malthus and Mill) were very much concerned with the conditions for this 'automatic growth'. These 'Classical'

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1 W.W.Rostow, "Rostow on Growth" in The Economist, August 1959, p. 409.
2 Ibid., p. 411.
3 Ibidem.

economists saw the development of a capitalist economy as a race between technological progress and population growth, in which they expected technological progress to be ahead for some time, but which would end in stagnation. Marx also expected capitalism to break down, not because of economic stagnation, but for sociological reasons. A sociological aspect is also essential in Schumpeter's theory: a certain 'social climate' is necessary if the entrepreneur is to play his vital role as 'innovator' in the process of capitalist growth. Marx noticed that economic development under capitalism tends to take the form of fluctuations; Schumpeter's main point was that the capitalist process is necessarily cyclical.

The economists occupying themselves with growth were relatively few in number. The major concern of economic science was to build up a static model of the economy. The economic system in stationary equilibrium was perfectly explained; and all the aberrations from this perfectly harmonious picture, such as unemployment or depressions, were regarded as temporary adjustments which would, if but left completely alone, disappear again.However, the terrible 'aberration' of the 30's made the economists aware that a more sophisticated theory was necessary to be able to explain such a phenomenon. Or was this phenomenon more than a temporary depression; was it the beginning of that so often heralded 'stagnation'?

This period, which in Rostow's framework marked the transition of the first mature economy into the stage of high mass consumption, caused the attention of the economists to turn to growth again.

Hansen saw a slackening off in the rate of population growth as the alarming cause for stagnation of the advanced economies. He thought that technological progress and resource discovery would not be sufficiently high to offset a diminishing population growth. Thus where the Classicals were afraid of population growth, Hansen considered it essential.

Harrod tried to develop a dynamic growth theory in contrast to the "active, but unchanging process"[1], with which economic theory had been occupied before. The trade cycle in his view became completely conditioned by its occurrence in a dynamic (i.e. growing) economy. In this thesis a study is presented of the literature that followed in the wake of Harrod's attempt at a truly dynamic treatment of the economic system.

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1 R.F.Harrod, Towards A Dynamic Economics, (Macmillan & Co, London, 1948); to be called 'Towards'; p. 4.

CHAPTER I

HARROD'S GROWTH MODEL.[1]

> "It is this rate of change, even more than the change itself, that I see as the dominant fact of our time" - J.A. Stratton, President, M.I.T. Centennial Address

Postulates.

In the development of a dynamic growth theory, Harrod defines 'dynamic'as"referring to propositions in which a rate of growth appears as an unknown variable."[2] Although appreciating the importance of lag analysis, Harrod purposely neglects lags in his analysis since he considers equations with 'rate of growth' variables far more fundamental than those expressing lags of adjustment. Harrod does not regard lags as in essence dynamic phenomena.

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1 R.F.Harrod, "An Essay in Dynamic Theory" in The Economic Journal, Vol. 49, 1939, pp. 14-33, to be called 'Essay'; and R.F. Harrod, Towards, Lecture 3, pp. 63-100.

2 Essay, p. 17.

Harrod based his dynamic theory on the following postulates:

"(1) that the level of a community's income is the most important determinant of its supply of saving;

(2) that the rate of increase of its income is an important determinant of its demand for saving, and

(3) that demand is equal to supply."[1]

To avoid misunderstanding it is noted that the concepts income, output, savings and investment have the dimension (money)value per unit of time.

With the following symbols:

Y- the national (or community's) income or output,

S- the national savings: the supply of net investment funds,

I- the national net investment: the demand for investment funds or savings; net investment does not include the investment expenditure for replacements,

s- the fraction of the national income saved: $s = \frac{S}{Y}$; of course, in a formula of this type S and Y must have reference to the same time period,

v- the investment or acceleration coefficient as defined by relation (2) below,

t- time,

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[1] Harrod, Essay, p. 14.

Harrod's postulates can be written:

(1) $S = f(Y) = sY$;

(2) $I = f(\frac{dY}{dt}) = v\frac{dY}{dt}$; this relation, which states that investment is induced by the rate of chango in output, is called the acceleration principle; v is the investment or acceleration coefficient;

(3) $I = S$; this condition expresses the dynamic equilibrium.

Hence:

If s and v are constants and if the value of Y at time zero is called Y_0, integration gives: $Y = Y_0 e^{\frac{s}{v}t}$. This solution represents the equilibrium path of Y over time.

The Actual Rate of Growth: G.

Let G be the actual rate of growth of income (or output); in Harrod's words: "the rate of increase in total output which actually occurs."[1] It is noted that the term 'growth' (which would be $\frac{dY}{dt}$) is commonly substituted for the more accurate term 'relative growth' ($\frac{1}{Y}\frac{dY}{dt}$), and that the word 'increase' in Harrod's definition must also be taken in a relative sense.

Thus: $G = \frac{1}{Y}\frac{\Delta Y}{\Delta t}$, or in the notation of differential calculus: $G = \frac{1}{Y}\frac{dY}{dt}$.

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[1] Harrod, Essay, p. 18.

Hence, from the relations above it follows that: $G = \frac{s}{v}$ From the equation $Y = Y_o e^{\frac{s}{v}t}$, which can now be written $Y = Y_o e^{Gt}$, it is seen that G is the geometric rate of growth of income. "G is a quantity determined from time to time by trial and error, by the collective trials and errors of vast numbers of people."[1]

Harrod uses the notation[2]:

x_0 - the total output (or income) in period zero,

x_1 - the total output (or income) in period 1,

K - the capital (stock) in the economy at a moment of time,

C - the actual marginal capital-output ratio: $C = \frac{\Delta K}{\Delta Y}$

The quantity C is defined as "the value of the increment of capital per unit increment of output actually produced."[3] As the actual capital-output ratio, C can be regarded as an 'ex post' quantity[4] ; and C covers both fixed and circulating capital, both durable and non-durable and both producer and consumer goods. C, which equals $\frac{\Delta K (\$)}{\Delta Y (\$/\text{year})}$ thus has the dimension of time.

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1 Harrod, Towards, p. 86.

2 Harrod, Essay, p. 18; however C will be used as in Towards, instead of C_p which is used in the Essay.

3 Harrod, Essay, pp. 17, 18.

4 'Ex post' means actual, realized; its counterpart is 'ex ante': expected, planned, intended.

From the definitions: $I = \frac{dK}{dt} = C\frac{dY}{dt}$ or $C = v$, the actual capital-output ratio equals the investment or acceleration coefficient.

When Harrod writes: $G = \frac{x_1 - x_0}{x_0}$, he presupposes that the time in which x_1 is the (money) income and the time period in which a change in income $(x_1 - x_0)$ takes place are always the same: a quantity $\Delta t = 1$ can thus be omitted from the equation. This expression leads, with $S = I$ or $sx_0 = C(x_1 - x_0)$ (again divided by $\Delta t = 1$), to $G = \frac{s}{C} = \frac{s}{v}$, a result already arrived at above.

The Warranted Rate of Growth: G_w.

Let G_w stand for the warranted rate of growth of income (or output):

> "that over-all rate of advance (rate of growth of income) which if executed, will leave the entrepreneurs in a state of mind in which they are prepared to carry on a similar advance. Some may be dissatisfied and have to adjust upwards or downwards, but the ups and downs should balance out and, in the aggregate, progress in the current period should be equal to progress in the last preceding period."[1]

G_w is an unstable, moving, dynamic equilibrium, representing

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[1] Harrod, Towards, p. 82.

"the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance."[1]

It is the entrepreneurial equilibrium, an equilibrium in which unemployment of manpower is a distinct possibility.

"The decision by each entrepreneur to continue producing at the rate he has produced or to produce something more (than the rate at which production has been made) is no doubt determined both by the satisfactory or unsatisfactory character of the results of his previous decisions as experienced to date -a point upon which the lag analysis lays primary stress- and also by a reasonable prognostication of what is to come based on a survey of the particular markets."[2]

Let C_r stand for the value of the capital goods required by technological and other conditions (state of confidence, rate of interest, etc.) for the production of a unit increment of output; it is the required marginal capital-output ratio. C_r as

"the new capital required to sustain the output which will satisfy the demands for consumption arising out of consumers marginal addition to income"[3]

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1 Harrod, Essay, p. 22.
2 Harrod, Towards, p. 82.
3 Harrod, Towards, p. 83.

or from the entrepreneur's point of view as

"that addition to capital goods in any period, which producers regard as ideally suited to the (additional) output which they are undertaking in that period"[1]

is thus the marginal requirement for new capital. For convenience C_r can be regarded as an 'ex ante' quantity, although Harrod is not sure whether it does correspond to it.

Harrod's fundamental equation is $G_w = \frac{s}{C_r}$. Its derivation in Harrod's words runs as follows:

"If the value of the increment of stock of capital per unit increment of output which actually occurs, C, is equal to C_r, the amount of capital per unit increment of output required by technological and other conditions . . . . then clearly the increase which actually occurs is equal to the increase which is justified by the circumstances. This means that, since C includes all goods . . . . , and is in fact production minus consumption per unit increment of output during the period, the sum of decisions to produce, to which G gives expression, are on balance justified -i.e., if $C_r = C$, then $G = G_w$ and . . . $G_w = \frac{s}{C_r}$."[2]

It should be noted that so far it has been implicitly assumed that s is an 'ex post' quantity. In this last relationship, however, s has an 'ex ante' character.

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[1] Harrod, Essay, p. 19.
[2] Ibid., p. 18.

Harrod uses only one symbol, but does make the distinction.

Thus G_w is uniquely determined by the propensity to save (the word 'propensity' implies the ex ante character of s) and the required quantity of capital per unit increment of total output. However, there is no unique warranted rate of growth: its value depends upon the two determinants s and C_r, whose values change in the course of the trade cycle with the level of activity.

The rather special optimum rate of growth of income where the entrepreneurial equilibrium obtains full employment of labour is called 'the proper warranted rate of growth of income' by Harrod.

Refinements.

The acceleration principle states that investment is induced by the rate of change in output, thereby emphasizing the short-period influence of changes in output. Other 'investment relations', relations which describe the dependence of investment upon output, are possible. Harrod realizes that a more realistic investment relation will be an important refinement, although he does not consider it essential for his pure theoretical growth model. Much of the outlay of capital -the investment- is connected with long-range planning; it is related to a prospective long-period increase of activity, or it is induced by new inventions. Long-range investment is somewhat influenced by the present state of prosperity. A high current level of income will

induce more investment than a low level of income. And the longer the current level of income has been in existence, the greater its influence will be on the business expectations. This effect contributes an amount kY to the investment, if its influence is assumed to be directly proportional to Y, with proportionality constant k. Another part of the long-range investment will be totally autonomous: independent both of the current level of income and of its current rate of change; this part of the total (net) investment can be called K. Adding these refinements to the original postulate gives: $I = v\frac{dY}{dt} + kY + K$

Harrod writes in his notation (S=I):

$sx_o = C(x_1 - x_o) + kx_o + K,$

where x_o, x_1, s and C are as defined on pages 5 and 7.

Hence: $G = \frac{x_1 - x_o}{x_o} = \frac{s - k - \frac{K}{x_o}}{x_o}.$

With G_w and C_r defined as before, a reasoning analogous to the one set forth on page 10 leads to the more refined fundamental equation: $G_w = \frac{s - k - \frac{K}{x_o}}{x_o}$.[1]

The interesting conclusion is that the larger the volume of autonomous investment and the larger the volume of investment dependent upon the level of output, the smaller will be the warranted rate of growth, ceteris paribus. And, when the amount required for the long-range capital outlay is equal to the amount of saving, the

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[1] Harrod, Essay, p. 27.

warranted rate of growth will be zero. Baumol [1], who made this observation, also observed that autonomous investment has an accelerating influence on the warranted rate of growth of income, since $\frac{k}{x_o}$ varies inversely with the level of income x_o, while the other term $(s - k)$ is a constant. The warranted rate of growth of income will not have a constant value as before, but will increase with time.

For an even more refined and realistic warranted rate of growth of income, it is possible to open the economy, i.e. to include the foreign trade factor; however, for his pure growth model Harrod stayed within the closed economy. In short, including the foreign trade factor can be done as follows. If i stands for the fraction of income spent on imports, and if E stands for the capital outlay on exports, -including the invisible exports and the foreign investments- , as independent both of the level of output and of its growth at home, then Harrod finds : $sx_o + ix_o = C(x_1 - x_o) + kx_o + K + E$; and from this Harrod derives: $G_w = \frac{s + i - k - \frac{K}{x_o} - \frac{E}{x_o}}{C_r}$ [2]; the difference between i and $\frac{E}{x_o}$ represents an international movement of capital.

Do Harrod's postulates cover his conclusions? It is certainly true that Harrod's postulates were not all explicitly stated; several were implicitly assumed and were hidden in some of his definitions and remarks.

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[1] W.J.Baumol, "Notes on Some Dynamic Models" in The Economic Journal, Vol. 58, 1948, pp. 506-521; to be called 'Notes'; p. 509.

[2] Harrod, Essay, p. 28.

Harrod's assumption in regard to the nature of producers' plans and expectations is implicit in his definition of the warranted rate of growth: "that over-all rate of advance which if executed, will leave entrepreneurs in a state of mind in which they are prepared to carry on a similar advance."[1] This means in short that Harrod assumes the coefficient of expectations (η) to be unity, whereby is meant that the entrepreneurs expect the rate of change in output to continue[2]. This is a quite realistic assumption, although it need not necessarily be so.

Alexander[3] especially blames Harrod for deriving too specific conclusions from his postulates[4]. According to Alexander it is again Harrod's assumption in regard to the entrepreneurial expectations that is omitted, and in order to remedy this shortcoming, Alexander wants to add the postulate:

> "If investment ex post is justified in any period, entrepreneurs will (unless prevented by physical limitations) in the succeeding period increase production in the same proportion as it has just been increased."[5]

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1 Harrod, Towards, p. 82.
2 The 'coefficient of expectations' was introduced by L.A.Metzler in his article: "The Nature and Stability of Inventory Cycles" in Review of Economics and Statistics, Vol. 23, 1941, pp. 113-129.
3 S.S.Alexander, "Mr. Harrod's Dynamic Model" in The Economic Journal, Vol. 60, 1950, pp. 724-739; to be called 'Model'.
4 Ibid., p. 728.
5 Ibidem.

However, this postulate states the same thing as Harrod does in the definition of his warranted rate of growth of output. Harrod does not mention in any way that the entrepreneurs may have different expectations; it is Baumol who considers the implications of this possibility; this will be discussed later in this chapter.

It is also important to realize for a better understanding of Harrod's fundamental equation $G_w = \frac{s}{C_r}$ that it is based on the following two implicit assumptions:

"(1) that inventions are neutral and

(2) that the rate of interest is constant."[1]

Since changes in technique may alter the ratio of capital to output, Harrod divides inventions and improvements into neutral, capital-using and capital-saving, according as they cause the ratio of capital to output, at a constant rate of interest, to remain unchanged, to increase or to diminish. By neutral technical progress is meant that the inventions and improvements taken as a whole are neutral.

It thus follows that an important refinement of Harrod's fundamental equation consists in dropping the assumption of neutral technical progress. If inventions are not neutral, let in Harrod's terminology "d (deepening) stand for the value of new capital installations during

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[1] Harrod, <u>Towards</u>, p. 83.

the unit period, expressed for convenience as a fraction of income."[1] Introduction of 'd' makes it possible to let C_r remain the required marginal capital-output ratio based on the assumption of neutral technical progress; and the same holds for C. This is explained in the following.

In order to avoid confusion it is opportune to distinguish, for the time being, between C_{rn} (C_r neutral), and the true C_r: C_{rt}. If the assumption of neutral technical progress is valid: $C_{rt} = C_{rn}$. C_r (i.e. C_{rn}) will then be

> "segregated as that capital requirement which essentially belongs to the growth of output as such, from the requirement for increased capital per unit (increment) of output."[1]

If now inventions are not neutral, but capital-using for example, the required marginal capital-output ratio will become C_{rt}, larger than C_{rn}; investment will then be necessary in order to increase the capital-output ratio from its constant value C_{rn} to its higher value C_{rt}, due to the capital-using invention.

From S = I, it follows now that

$sx_o = C(x_1 - x_o) + dx_o$,

dx_o is the necessary extra investment as required by the capital-using invention; d is expressed as a fraction of income. The usual derivation leads Harrod to:

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[1] Harrod, Towards, p. 96.

$G_w = \frac{s-d}{C_r}$, where C_r is still C_{rn}, so that in fact $C_{rt} = \frac{s}{s-d} C_r$ $(= \frac{s}{s-d} C_{rn})$, because $G_w = \frac{s-d}{C_{rn}} = \frac{s}{C_{rt}}$, and indeed, when d = 0, $C_{rt} = C_{rn}$, i.e., d equals zero represents the case of neutral technical progress. Naturally, if the inventions are capital-saving, d will be negative.

Dropping the second implicit assumption of a constant rate of interest forms another interesting refinement of G_w. An adjustment similar to the non-neutral inventions can be made. C_r remains again the capital-output ratio belonging to a constant rate of interest and if now the rate of interest falls for example, the demand for savings -i.e. investment- will increase as a result. The extra investment can be expressed as $d.x_o$[1], where d is again a fraction of income, and Harrod ends up again with $G_w = \frac{s-d}{C_r}$. With an increasing rate of interest d will be negative. If the rate of interest would keep falling, investment would keep increasing; thereby increasing the required marginal capital-output ratio of the whole economy. And according to Harrod: "If d is positive, C_r (i.e. C_{rt}) will increase through time, and may eventually become so great as to enable us to dispense with d. At that point interest need fall no further."[2]

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1 This is a different d from the one used above.
2 Harrod, Towards, p. 96.

<u>The Relationship between G and G_w : Harrod's Cycle Theory.</u>

What does inequality between G and G_w entail? What are the implications of a departure of the actual rate of growth from the warranted rate of growth?

> "Suppose an excessive (rate of growth of) output, so that G exceeds G_w. The consequence will be that C, the actual increase of capital goods per unit increment of output, falls below C_r, that which is desired. There will be, in fact, an undue depletion of stock or shortage of equipment, and the system will be stimulated to further expansion. G, instead of returning to G_w, will move farther from it in an upward direction, and the farther it diverges, the greater the stimulus to expansion will be. Similarly, if G falls below G_w, there will be a redundance of capital goods, and a depressing influence will be exerted; this will cause a further divergence and a still stronger depressing influence; and so on."[1]

Thus: on either side of the path of the warranted rate of growth

> "is a 'field' in which centrifugal forces operate, the magnitude of which varies directly as the distance of any point in it from the warranted line. Departure from the warranted line sets up an inducement to depart farther from it."[2]

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1 Harrod, <u>Essay</u>, p. 22.
2 <u>Ibid.</u>, p. 23.

This instability principle is based on the assumption that at least some current orders depend on the rate of change in the entrepreneur's turnover. Since each individual firm is definitely somewhat influenced by current experience, the instability principle will operate: a deviation upwards from the warranted rate of growth causes a shortage of fixed and / or circulating capital in relation to current requirements. On the overall average firms faced by a novel shortage will place more orders than they otherwise would, causing a further deviation from the path of warranted growth.

A shortage of capital goods ($C < C_r$) is thus the consequence of production above the warranted rate of growth of output ($G > G_w$). Production, here, is meant not as a level of output, but as a rate of growth of output. Conversely, too much investment, more investment than required at that moment ($C > C_r$), is caused by production below the warranted level ($G < G_w$). In the above comparison of G and G_w, only one s has been used. In fact, the possible divergencies of ex post from ex ante saving will have the same effect on the rate of growth as the divergences of C from C_r. However, since Harrod sees no possible causes for a divergence of ex post from ex ante saving, he more or less assumes that they will be equal and thereby he implicitly assumes that all the adjustments will take place through investment.[1] In the following this same

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[1] Harrod, Essay, p. 20.

assumption will still be made.

Does not the doctrine of instability -the principle that there are 'centrifugal forces' active on either side of the warranted rate of growth- have to be modified? That is are the values of s and C_r independent of the value of G? Will not a change in s or C_r occur in the reaction interval as a result of a change in G? In particular will not s and C_r change in such a fashion that G_w will increase or decrease, so that the gap is closed instead of widened?

In the case of an increase in G above G_w, it seems more likely that as a result of the higher actual output, C_r will be raised, than that it will be reduced. This can be explained as follows. The actual C goes down, i.e. the investment productivity[1] (the marginal efficiency of capital) goes up. The entrepreneur is therefore prepared to pay more for his capital; he is prepared to let his investment increase relatively more than he wants his output to increase. In other words, C_r, if anything, will go up. G_w then will not increase and follow G; contrarily so, the gap between G and G_w will widen even more. And vice versa for the case of a fall in G below G_w ; in a phase of declining rate of growth the required capital-output ratio is not in general likely to rise, so G_w is

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[1] The investment productivity is the reverse of the capital-output ratio; this concept will be further introduced in chapter II.

not likely to fall; the gap, again, will not be closed.

And how will the propensity to save react to a change in G? An increase in G above G_w might result in an increase of ex ante s due to the expansion of activity. The accompanying increase in ex post s will quite likely be less, so that, due to this effect G_w will increase more than G, and the gap will tend to close. In the opposite case, the opposite effect will hold. But will this behaviour of the propensity to save be a significant stabilizing factor; will it modify the instability principle considerably? The influence which a high marginal savings coefficient will have on the total s will be small when G is still in the neighbourhood of G_w, since the additional income will still be relatively small. This stabilizing influence must therefore be considered insignificant near the equilibrium G_w: the instability principle will be secure, by whatever amount the proportion of income saved will increase as a result of the expansion of activity. However, when discussing the factors which limit the increase of G near the 'saturation point', considerable importance will be given to this 'ceiling' effect. Thus it has been shown that C_r and s fail to act as stabilizers.

Two outside stabilizers of the instability principle have been suggested. Baumol [1] contends that a

[1] Baumol, Notes, pp. 512, 513.

return of the rate of increase (or decrease) of output to the warranted rate of growth may be possible if a pattern of producers' plans and expectations exists, different from that assumed by Harrod. If attempting to describe this numerically, one might think of the coefficient of expectations η.[1] Metzler defined the concept as the ratio of the change expected by the producers in output between periods t and (t + 1) over the observed change in output between periods (t - 1) and t. This definition gives the producers a memory which is short (one period) and fixed. Rigid use of the formula can therefore easily lead to an unrealistic behaviour. Another disadvantage of η is that its value would have to change; the need to describe the aspects of these changes would merely shift the problem from describing the results of a certain η to its cause. Another definition of η, one which would not have these disadvantages, does not present itself. It is to be noted however, that the value $\eta = 1$ (a given rate of change is expected to continue undiminished) does have a useful interpretation; as mentioned before[1] it corresponds to Harrod's implicit assumption with respect to the producers' expectations and it leads to the instability principle. Baumol's contention is that producers could have a different expectation; in case they expect a deviation

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[1] See also p. 14 of this chapter.

from the warranted path to be merely a temporary phenomenon, Harrod's system may be inherently stable. It is uncertain which pattern of expectations is more likely to prevail in reality; it will depend upon the particular circumstances what expectations the entrepreneurs will have, and these will vary. Harrod's assumption of $\eta = 1$ is probably quite realistic in many cases, and thus his unstable system may be quite probable.

The other stabilizing effect may be found in a monetary aspect: a rise in the general price level. According to Alexander[1] "gently rising prices may act to stabilize a moderate rate of growth that would otherwise develop into an excessive one."[2] Increased expenditure may not only call forth increased production, it may also cause prices to rise if there is some price flexibility. Buyers can react to the price increase in essentially two ways: in the first place, they can spend as if the price level had not changed. 'Forced savings' result: the buyers get less for their money than they intended to buy. Real output (income) will thus decrease, or more precise in terms of a growing economy: the increase in real income associated with the increase in expenditure will be reduced as a result of the price increase. Consequently there will be less induced investment. This reaction then would normally act as a

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[1] Alexander, Model, pp. 737- 739; and also S.S.Alexander, "The Accelerator as a Generator of Steady Growth" in Quarterly Journal of Economics, Vol. 63, May 1949, to be called 'Accelerator', pp. 175, 176.

[2] Alexander, Accelerator, p. 175.

stabilizer; however if the price increases are too high, speculative investment may be induced by the rising prices, thereby impairing the stabilizing effect. In the second place, buyers may increase their expenditures in proportion to the rising prices; the price rise will then be without a stabilizing effect. If the money supply permits this, an inflationary process will result.

Some economists find a nonmonetary treatment of the growth problems unacceptable; for example, according to Yeager, "nothing in the growth economics of Harrod and Domar has revealed forces in the real world that tend, quite apart from the behaviour and management of money, to produce depressions and inflationary booms."[1] Nevertheless, not denying that of course monetary aspects also help to shape the economic system in reality, a nonmonetary explanation of the essential growth factors seems far more suitable for a better understanding of the working of the purely economic factors. Therefore, not only in Harrod's model, but in all the following models also, the monetary factors in the economic system are neglected.

The tendency of G to run away from G_W, -the instability principle- ,thus remains a very basic factor at the heart of the trade cycle problem. The various phases of the trade cycle will now be discussed.

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[1] L.B.Yeager, "Some Questions about Growth Economics" in American Economic Review, Vol. 44, 1954, p. 63.

In the upward phase of the cycle the actual rate of growth will tend to run above the warranted rate ($G > G_w$) and thus the accretion of capital will be less than follows from the required acceleration coefficient ($C < C_r$); and conversely in the downward phase. The volume of investment fluctuates less than would follow from the theoretical value C_r of the acceleration coefficient. This fact was known earlier from statistical investigations.

Although ex post saving as a fraction of income is fairly steady in the long run, ex ante saving is not likely to be steady, especially not in the short run. During the course of advance, when $G > G_w$, companies very likely plan to save a large fraction of the short period increases of the net receipts. However, due to the shortage of capital goods ($C < C_r$), the companies will invest more and their ex post savings will turn out to be less than their planned savings. Similarly consumers will plan to save a lot in good times. In the beginning of the upswing their expenditures will usually adapt only slowly to their increasing incomes; but as the good times continue, the optimistic tendency of the times will result in less actual savings than were planned. Thus in the upward phase of the cycle G_w will rise and sooner or later, G_w will rise so much that G will fall below it, thereby precipitating the spiral of depression.

Another important reason for the downturn is the decreasing mobility of the factors of production. Owing to

the increasing difficulties of transferring labour and other resources to the more desired uses, G will be reduced in the later stages of the advance. The downturn will thus come about before the true ceiling of total employment of labour and the other resources has been reached.

And thus through the total of accumulating forces G will eventually depart downward from G_w : $G < G_w$: the depression has set in. There is now much redundant capacity ($C > C_r$) and C_r will thus have a very low value for a while. However, in the depression the ex post savings will eventually turn out to be larger than the ex ante savings, expressly because there are hardly any suitable investment outlets due to the redundant capacity still available. Ex ante s will fall below ex post s, thereby lowering G_w relative to G. So eventually the forces of revival will set in again.

Thus

"as actual growth departs upwards or downwards from the warranted level, the warranted rate itself moves, and may chase the actual rate in either direction. The maximum rates of advance or recession may be expected to occur at the moment when the chase is succesful."[1]

"It is the departures from G_w, not the value of G_w itself, which have paramount influence in producing boom and slump."[2]

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1 Harrod, Essay, pp. 28, 29.
2 Harrod, Towards, p. 88.

Summarizing the relationship between G and G_w, it can be said that

> "the movement of a dynamic determinant (savings, the acceleration coefficient, etc.) has an opposite effect on the warranted path of growth to that which it has on its actual path."[1]

According to Harrod, changes in fundamental conditions have opposite effects on the actual rate and the warranted rate: this is the dynamic principle.

The Natural Rate of Growth: G_n.

Let G_n stand for the natural rate of growth of income (or output):

> "the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work / leisure preference schedule, supposing that there is always full employment in some sense."[2]

The economic system cannot normally advance more quickly than the natural rate allows. "G_n sets a limit to the maximum average value of G over a long period."[3] Only after a recession G may attain a higher value than G_n for a considerable period.

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1 Harrod, Essay, p. 23.
2 Ibid., p. 30.
3 Harrod, Towards, p. 87.

The Relationship between G_w and G_n:

Harrod's Trend Theory.

Will it be better to have the 'proper' warranted rate of growth above or below the natural rate of growth? It should be remembered that the warranted rate of growth and the 'proper' warranted rate of growth are entrepreneurial equilibria; they are wanted equilibria by the entrepreneurs, but they will not necessarily be realized. If the natural rate of growth -the maximum potential rate of growth of the economy at a certain moment of time- is below them, they will not be able to become realized. So, although it seems strange that Harrod bases his trend theory on the comparison of the natural rate of growth with the proper warranted rate of growth and not on a comparison with the warranted rate of growth as such, it does not make an essential difference.

If the proper warranted rate is above the natural rate, there will be a chronic tendency to depression;

> "the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment."[1]

This forms the problem of chronic unemployment.

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[1] Harrod, Essay, p. 30.

"If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profit booms."[1]

So it may be better to have the proper warranted rate lower rather than higher than the natural rate.

And if the proper warranted rate is tending to be above the natural rate, it may be very healthy for the economy to have its proper warranted rate reduced. Thus the long-run value of a stimulant can only be assessed if it is known whether, in its absence, the proper warranted rate is running above or below the natural rate. Thus

"the relation of G_n to G_w is clearly of crucial importance in determining whether the economy over a term of years is likely to be preponderatingly lively or depressed."[2]

This provides the most fundamental explanation

"of the common view that it is dangerous for an old country to be a large importer of capital. For this involves a high warranted rate of growth, and it is dangerous to have a high warranted rate when the natural rate is low. Per contra for a young country, whose natural rate is high, it is considered healthy and proper to have a large import of capital."[3]

The paradox of 'progress through thrift, as

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1 Harrod, Essay, p. 30.
2 Harrod, Towards, pp. 87, 88.
3 Harrod, Essay, p. 31.

advocated by the Classicists versus Keynes' 'prosperity by spending' slogan can now also be much better understood. A higher propensity to save warrants a higher rate of growth; this is not only all right, but desirable, if the warranted rate of growth is below the natural rate, as in the case of the underdeveloped, young economies and in the case of England in the time of the Classicists. But in the mature economies (Keynes' England) the natural rate of growth is low, especially because of the decline in population growth; and if the warranted rate then exceeds the natural rate, it is only too desirable to lower the propensity to save.

With regard to policy,

> "the nature of the measures suitable for combating the tendency to oscillate may depend on whether the natural rate is above or below the proper warranted rate. In addition to dealing with the tendency to oscillation when it occurs, it may be desirable to have a long-range policy designed to influence the relation between the proper warranted rate of growth and the natural rate."[1]

> "The ideal policy would be to manipulate the proper warranted rate so that it should be equal to the natural rate."[2]

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1 Harrod, Essay, pp. 31, 32.
2 Ibid., p. 32.

CHAPTER II

DOMAR'S GROWTH MODEL[1]

Purpose.

Domar's purpose was to improve upon Keynes' treatment of investment. In the short-run Keynes treated investment as an income-generating instrument, while ignoring its effect on the productive capacity (of the capital stock); and in his long-run analysis investment served to augment the stock of capital, while its income-generating (multiplier) effect was ignored. Domar wanted to narrow this gap which was left in the theory of income and employment by recognizing both attributes of investment by means of a simple (differential) equation,

> "the solution of which yields the rate of growth of investment and / or of national income that is required to keep the two effects of investment in balance."[2]

Thus Domar wanted to show that there exists an equilibrium rate of growth of income, which will not lead to the diminishing profit rates, scarcity of investment opportunities and chronic unemployment, which writers through the ages

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1 E.D.Domar, Essays in the Theory of Economic Growth (Oxford University Press, New York, 1957); specifically: "The Foreword", pp. 3- 15; Essay I: "A Theoretical Analysis of Economic Growth" (1952), pp. 16- 34; Essay III: "Capital Expansion, Rate of Growth, and Employment" (1946), pp. 70- 82; Essay IV: "Expansion and Employment" (1947), pp. 83- 108; Essay V: "The Problem of Capital Accumulation" (1948), pp. 109- 128.

2 Ibid., p. 7.

have expected as the effects of capital accumulation resulting from investment. The economy will be said to be in equilibrium when it grows in a certain way and the business cycle then becomes a deviation of the economy from its equilibrium rate of growth.

Assumptions and Definitions.

Domar made for his model the following explicit assumptions:

a that the economy is a private capitalist one, in which the government plays a minor role;

b that there is a constant general price level;

c that the relative distribution of income between labour and capital remains unchanged;

d that small changes in the interest rate do not significantly stimulate investment, i.e. that the interest rate can be assumed to be constant;

e that saving includes undistributed corporate profits;

f that saving and investment are 'net', i.e. do not include depreciation and similar charges;

g that depreciation is measured by the cost of replacing the depreciated asset by another one of the same productive capacity, thus not by historical costs;

h that there is no spontaneous (autonomous) investment;

i that there are no lags, thus for example saving in a certain period results from the income of the same period;

"for secular problems these lags are not likely to be of great importance, but they may play an essential role over the cycle"[1];

j that the marginal propensity to save equals the average propensity to save (α);

k that the productive capacity of an asset or of the whole economy (in which case the symbol P is used) is a measurable concept; defined as: "its total output when its labor force is fully employed in some conventional sense"[2], given the whole complex of economic and institutional conditions; the productive capacity of a country is a function of its natural resources, its labour force, its capital stock and its state of technique (the determinants of Harrod's natural rate of growth)[3];

l that employment (N) is a function of the ratio of national income to productive capacity, more particularly $N = Y/P$; thereby not distinguishing between idle machines and idle men[4];

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1 Domar, Op. Cit., p. 96.
2 Ibid., p. 71.
3 Domar must have been preoccupied with the problem of labour unemployment, since it would seem that he omits mentioning of full employment of capital and other non-labour resources in his definition of P.
4 It is, with regard to policy matters however, very important to distinguish between unemployed capital, which inhibits new investment and unemployed labour, which has the (political) effect of increasing investment. Apparently, the Keynesian doctrine of 'more spending to combat unemployment' has not yet been applied to the other factor of production: capital; of course, in practice unemployed capital and unemployed men go hand in hand.

m that the economy is in equilibrium when its productive capacity P equals its national income Y, or more simply: when N = 1, when there is full employment (of all resources).

Furthermore Domar introduces two other concepts:

1 s : "the ratio of the productive capacity net of depreciation (net value added) of the new projects to capital invested in them"[1].

Thus: $s = \frac{1}{I}\frac{dP'}{dt}$ [2], where P' is the net productive capacity of the new projects.

With $I = \frac{dK}{dt}$: $s = \frac{dP'}{dK}$ [3].

I times s is thus the net (annual) potential output of the (annually) new projects.

2 σ : "the potential social average investment productivity"[4], thus σ refers to the 'net' increase in potential productive capacity of the whole economy: $\sigma = \frac{1}{I}\frac{dP}{dt}$ [5]; σ indicates the increase in productive capacity which accompanies, rather than which is caused by each dollar invested.

Hence, s and σ are reciprocals of a marginal capital-output

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1 Domar, Op. Cit., p. 73.
2 The derivation of this equation is clear, when one realizes what the dimensions are of the different quantities (see chapter I, pp. 5,7, where also I and K are defined)
3 Domar, Op. Cit.,p. 77. Domar uses the expression s = P/K. From the definition of s it is clear, however, that he means a marginal quantity: properly, $s = \frac{\Delta P}{\Delta K}$ or $s = \frac{dP}{dK}$. Also, he mistakenly uses P, meaning the P' as defined above.
4 Ibid.,p. 74.
5 Ibidem.

ratio: s refers only to the new projects, while in σ also the possibility of a decreasing productivity in the older projects, due to the addition of the new projects, is included: σ is the algebraic sum of the two effects. Thus $\sigma \leqslant s$; this Domar thinks to be the normal case. The possibility $\sigma > s$ he considers to be a rare exception; apparently he only considers the case of secular stagnation.

<u>The Required or Equilibrium Rate of Growth.</u>

> "Assuming that output and capacity are in balance at the start, under what conditions will this balance be preserved over time, or in other words, at what rate should they grow to avoid both inflation and unemployment?"[1]

By 'output' is meant the demand for the output of the economy, and by 'capacity' is meant the supply of output produced by the productive capacity of the economy. The balance thus means $Y = P$ or $N = 1$.

On the demand side for output are the different sectors of the economy: the government, whose demand (g) will be neglected (the government plays a minor role); the consumers, whose (consumption)demand C [2] will be regarded as a dependent variable (dependent on income); and the producers, whose (investment)demand I will be treated as

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[1] Domar, <u>Op. Cit.</u>, p. 19.
[2] It should be noted that Domar's C stands for consumption demand; Harrod's C for the actual marginal capital-output ratio.

the active, independent variable: "the rate of growth of demand is a function of the rate of growth of investment"[1].

Thus : $Y = C(Y) + I$, and

$\frac{dY}{dt} = f\left(\frac{dI}{dt}\right)$; the dependence of Y on I constitutes the income-generating or multiplier effect of investment.

With : c - the marginal consumption coefficient,

α - the marginal savings coefficient, $(\alpha + c = 1)$,

whereby both α and c are either ex post or ex ante concepts (Domar does not make the distinction; α and c will normally be regarded as ex ante concepts, i.e. as propensities) :

$c = \frac{dC}{dY}$, or $\frac{dC}{dt} = c\,\frac{dY}{dt}$.

Differentiating $Y = C + I$ with respect to t gives:

$\frac{dY}{dt} = \frac{1}{\alpha}\frac{dI}{dt}$; $\frac{1}{\alpha}$ is called the multiplier.

The supply of output of the economy could best be described by a production function, but because of the complexities of such functions Domar prefers to regard the (annual) change in output-$\frac{dP}{dt}$ as a function of investment. The function is: σI. By multiplying I with its average productivity σ, which accounts for the growth of labour, the changing natural resources and technological progress, investment, and thus capital stock, is treated in this model as the only explicit factor of production.

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[1] Domar, Op. Cit., p. 22.

It was the traditional approach to take output as a function of labour input (manhours worked). This was then inflated by some increase in manhour productivity due to the movements of the other determining factors. Because labour is more homogeneous and more easily measurable this seemed to be a more practical approach. Theoretically, however, it represents only one side of the growth process: it only affects productive capacity without generating income, so there would be no common variable in the two sides of the equation P = Y. The dual nature of the investment process of both enlarging the productive capacity and generating income (investment increases productive capacity, but only the rate of increase in investment increases national income), makes the capital factor the ideal variable to use. Thus $\frac{dP}{dt} = I\sigma$ provides the supply side: the (annual) increase in output which the economy can produce.

Differentiation of the equilibrium equation P = Y with respect to time, and substitution of the derived relationships gives:

$$I\sigma = \frac{1}{\alpha}\frac{dI}{dt}.$$

Assuming α and σ to be constants -statistics seem to indicate that on the average the marginal investment productivity (σ) has not changed much in the western world over 80 years or so- integration gives: $I = I_0 e^{\alpha\sigma t}$, with the initial condition $I = I_0$ for $t = 0$.[1]

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[1] Domar, Op. Cit., p. 75.

Describing an (anticipated) equilibrium, S (ex ante savings) must equal (ex ante) I.

Therefore: $I = a\,Y$.[1]

Differentiating with respect to time, and dividing by I (=aY) gives:

$\frac{1}{I}\frac{dI}{dt} = \frac{1}{\alpha Y}\frac{\alpha\, dY}{dt} = \frac{1}{Y}\frac{dY}{dt}\ (=\alpha\sigma)$.[2]

Also from $I = sY = \alpha Y$: $Y = \frac{1}{\alpha} I_0 e^{\alpha\sigma t}$.

Thus the maintenance of full employment requires investment and income to grow at a constant annual relative rate of $\alpha\sigma$; $\alpha\sigma$ is the required[3] or equilibrium rate of growth. Neither Domar nor Harrod ever ventured on the economics of an accelerating -or decelerating- rate of growth of income.[4] Because Domar's assumption that the marginal and average propensity to save are equal, is $\frac{1}{I}\frac{dI}{dt} = \frac{1}{Y}\frac{dY}{dt} = \alpha\sigma$. Presumably Domar worked in terms of the increment of investment because he was anxious to show that investment must itself grow if the economy is to grow[5].

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1 The symbol $a = \frac{S}{Y}$ is used, where in Harrod's terminology s was used, because Domar has used s already in another meaning. Domar does not use a symbol for the average propensity to save.

2 The assumption a is constant can be made, but is not necessary, because it has already been assumed that α is constant.
Differentiating $a = \frac{S}{Y}$ = constant, gives: $a = \frac{dS}{dY} = \alpha$.
But integrating $\alpha = \frac{dS}{dY}$ = constant, gives $S = \alpha Y$ + (a constant) or $\alpha = a$, only with the additional initial condition: $Y = 0$, $S = 0$.
It is clear then, that the assumptions a = constant, α= constant and α = a (= constant) are for all practical purposes identical. The assumption s = constant has therefore not been restated explicitly.

3 It should be stressed that Domar's 'required' is a concept entirely different from Harrod's 'required'.

4 R.F.Harrod, "Domar and Dynamic Economics" in The Economic Journal, Vol. 69, 1959, pp. 454, 455; to be called 'Domar'.

5 Harrod, Domar, p. 454.

Harrod makes no reference whatsoever to $\frac{1}{I}\frac{dI}{dt}$ and only an implicit reference to I by his definition of C as $\frac{I}{\frac{dY}{dt}}$.[1] By defining G as $G = \frac{1}{Y}\frac{dY}{dt}$, he can then write: $G = \frac{1}{Y}\frac{dY}{dt} = \frac{\frac{dY}{dt}}{I/s} = \frac{s}{\frac{I}{\frac{dY}{dt}}} = \frac{s}{C}$: he does not need the assumption $\alpha = s$ ($\alpha = a$). Basically the assumption is necessary for writing a trend equation in I or Y (the integration needs a constant a or s), and to tie the rate of growth of investment to the rate of growth of income.

When investment grows at some constant percentage rate r, which is not necessarily equal to the equilibrium rate , it can be proved that, when t is large, $\vartheta = \frac{r}{\alpha\sigma}$ equals $N = \frac{Y}{P}$. In other words, ϑ,[2] the coefficient of utilization, equals N (employment).

Domar's and Harrod's Growth Rates.

The most likely interpretation of a comparison between Domar's growth rates and Harrod's growth rates seems to be:

Domar's r is Harrod's actual rate of growth G,

Domar's αs might be Harrod's warranted rate of growth G_w,

Domar's $\alpha\sigma$ might be Harrod's natural or potential rate of growth G_n.[3]

Harrod has a fourth rate of growth, the proper warranted rate of growth G_{pw}, for which Domar has no counterpart.

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1 Harrod, Domar, pp. 452, 453.
2 Domar, Op. Cit., pp. 76, 77.
3 Harrod, Domar, p. 456.

Some dispute is possible about the meaning of $\alpha\sigma$ in Harrod's terminology. It belongs to a full utilization of P, the productive capacity, which means that there is "full employment of labour in some conventional sense" (p. 33, k). In Domar's definition of N (p. 34, 1) he does not, however, distinguish between idle machines and idle men. Full employment of both labour and capital would there be the condition for N, P and thus $\alpha\sigma$. It is thus possible to interpret $\alpha\sigma$ as meaning Harrod's G_{pw}, or his G_n. The addition that labour should be fully employed in 'some conventional sense', however, seems decisive in excluding the possibility that G_{pw} is $\alpha\sigma$: the 'entrepreneurial sense' of G_{pw} is hardly a 'conventional sense'. It seems thus safe to assume that Domar means by $\alpha\sigma$ the natural or potential rate of growth G_n.

What do Domar and Harrod say on the subject themselves? Domar contends that "$\alpha\sigma$ indicates the rate of growth of income necessary for the maintenance of full employment of labour; αs indicates that needed for a full utilization of capital".[1] It is rather irritating that nothing is said of capital in the description of $\alpha\sigma$, because in this way it is not made any clearer whether $\alpha\sigma$ equals G_{pw} or whether $\alpha\sigma$ equals G_n. Full utilization of capital is what the entrepreneur wants, therefore only under the assumption that 'full utilization of capital' has the same meaning to Domar as it has to Harrod: $\alpha s = G_w$. The phrasing: αs might be G_w, is preferred.

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1 Domar, Op. Cit., p. 114.

Harrod seems to agree with the conclusions drawn above in reference[1], but in reference[2] he points out, while discussing Domar's model, that Domar designated "the potential increase of output per unit of new investment" by , while Harrod defined C_r as: "how many units of new investment are required, on the assumption that the new investment is properly utilised, to produce an extra unit of output." Thus, since σ is valued on the basis that the new investment is properly utilised and "C_r is valued on the basis that the new investment is no more nor less than that required to produce a growth of output, i.e., that it is properly utilised, it is evident that $\sigma = \frac{1}{C_r}$." C_r gives expression to the asceleration principle, about which Domar appears to say nothing. Thus subject to a few minor reservations, "Domar's equation is identical with mine."

Harrod's equation of σ and $\frac{1}{C_r}$ can not be correct, because σ belongs to full employment of labour, and C_r (G_w) does not. Harrod must either mean that $\alpha\sigma$ is the proper warranted rate of growth, or he writes σ, while meaning s. Of course, Harrod's 'proof' is based on the fact that in both rates of growth, investment is 'properly' utilised. This is a rather loose term, however, which, conceivably, has a different meaning to the entrepreneur or in 'some conventional sense': it seems hardly a basis for forming

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1 Harrod, <u>Domar</u>, p. 456.
2 <u>Ibid.</u>, p. 452.

an opinion. In both $\alpha\sigma$ and G_n, capital is 'properly utilised'. The reasons why we prefer this interpretation of the meaning of $\alpha\sigma$ are set out above.

<u>The Junking Process: $\sigma < s$.</u>

Investment in new projects yields an (annual) productive capacity of Is; however, the productive capacity of the whole economy increases only by I . The productivity of total investment may be affected by shifts from existing to new projects of labour and of other factors of production. Thus somewhere in the economy productive capacity is reduced by an amount $I(s-\sigma)$; thus every year an amount of capital stock equal to $\frac{I(s-\sigma)}{s}$ becomes useless[1].

$\frac{I(s-\sigma)}{s}$ can either be regarded as capital loss (capital losses are not taken into account in calculating income and investment), or it can be treated as a special allowance for obsolescence. In the first case capital has to be redefined as the integral of investment minus capital losses[2]: thus every year chunks of capital, over and above depreciation, are written off and junked. In the second case net investment is $I - \frac{I(s-\sigma)}{s} = \frac{I\sigma}{s}$.[3]

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1 Domar, <u>Op. Cit.</u>, p. 77.
2 On p. 34 it was noted that Domar uses the expressions $s = \frac{P}{K}$ and $\sigma = \frac{P}{K}$, but that he should have used $s = \frac{dP'}{dK}$ and $\sigma = \frac{dP}{dK}$. To suit the meaning of s, P was redefined and became P'; K remained the capital stock in the sense of Harrod's definition, which is the 'generally' accepted sense. Here, however, Domar redefines K and not P: using symbols he could have said $s = \frac{dP}{dK'}$, as against $s = \frac{dP'}{dK}$.
3 Domar, <u>Op. Cit.</u>, p. 78.

The construction of new investment projects makes certain assets useless. Thus, for example, old plants reduce their output because their markets are captured by new plants; or new plants are not operated to capacity because they are unable to find a market for their products. In this case capital is idle[1] because $\sigma < s$. The junking process is an integral part of a free enterprise dynamic economy, in which investors make errors and thus misdirected investment; it is a relatively permanent phenomenon. No correction through a larger expenditure will be possible in principle, although with a growing income there will be more place for everyone.

The process thus described seems to contradict earlier statements: the growth rate $\alpha\sigma$ was Harrod's Gn, in which there was no idle capital. The described process must be seen as a short-term effect; in the long run adjustments are supposed to be made. When adjustments are made, idle capital is junked and therefore does not exist, i.e. s is equal to σ. When adjustments occur instantaneously, s becomes instantaneously equal to σ ;the two can not be separated, s in fact does not exist. The investment of an individual entrepreneur may have a productivity different from σ, but for the economy as a whole there is only σ.

The junking effect, by making old plants 'prematurely' obsolete, can, in some cases stimulate new

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1 This is not to say that other resources can not be idle also.

investment in new parts of the economy (in new pioneering ventures). Consequently, excess capacity in some industries does not have to be ruinous for an overall growth of output: the excess capacity and the new investment may occur in different sectors of the economy and are then being experienced by different people.

As a rule, however, the presence of idle capital will inhibit new investment. The relative importance of these two effects depends on the structure of the industry and of the economy in general. The effect of unused capacity on new investment will be smaller, when the industry is more atomistic, when competition in the economy is more fierce or when investment is more influenced by territorial and other changes (shifts in consumer preferences for example). The junking process can then be speeded up without a depressing effect. In this reasoning monopolized or connected industries are a threat to new stimulating ventures.

Besides idleness of capital caused by $\sigma < s$, capital can also be idle because $r < \alpha\sigma$. When the actual rate of growth r is smaller than the equilibrium rate of growth $\alpha\sigma$, there is too much investment relative to the (annual) change in output required by the changes in market demand (the marginal capital-output ratio is too large), i.e. there is an insufficiency of effective demand. In actual manifestation: cost-price squeeze, there is no difference between the causes of idle capital, but the distinction between them is very important for practical policy. This case of $r < \alpha\sigma$, is,

according to Domar[1], only a temporary phenomenon. It need only be a temporary phenomenon, because it can be corrected by proper fiscal and monetary policies.

Domar comes to the conclusion that even

> "if the economy fails to grow at the required rate, the relative disparity between its capacity and income does not become wider, because its capital also grows, not at the rate $\alpha\sigma$, but at the rate r." [2]

This is Harrod's case of $G < G_w$, but his conclusion, as opposed to Domar's, is an ever-widening gap between capacity and income. Harrod, believing in his centrifugal forces, does not conclude that this case is only a temporary phenomenon; according to his theory a downward spiral of depression will be the result. Domar probably thinks that the other factors that determine the productive capacity will catch up, but this will probably not happen until income has made a downward plunge; unless indeed monetary and fiscal policies have been set to work effectively.

However, Harrod apparently is under the impression that "Domar broadly endorses the instability principle".[3] Granted that this is so; then if the junking process has a discouraging effect on the propensity to invest, the reduction in investment will cause growth to fall below the level $\frac{1}{I}\frac{dI}{dt} = \frac{1}{Y}\frac{dY}{dt} = \alpha\sigma$. Thus the excess of

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1 Domar, Op. Cit., p. 79.
2 Ibid., p. 77.
3 Harrod, Domar, p. 458.

s over σ will, by leading to junking, cause the actual rate of growth r to fall below both αs and $\alpha\sigma$. In Harrod's terminology the fact that G_w (the rate of growth required to absorb saving) is above G_n (the rate of growth required to maintain full employment, having regard to technological progress) will cause the actual rate of growth to fall below either. Thus the two theories of $s > \sigma$ and $G_w > G_n$ are very near and express the same problem.[1]

According to Harrod[2], Domar does not even seem to realize that a high rate of junking might be bad from a purely productivity point of view, since there is in fact an optimum rate of junking. To explain Harrod's point, it is necessary to look more closely at Domar's mechanism. Domar seems to think that for a period, presumably in a spirit of buoyancy, as much new capital equipment will be created as is permitted by the savings available, regardless of the profitability. Then if there is too much capital equipment, the old equipment will be junked. That this tends to increase the growth rate (which is lower than $\alpha\sigma$) can be demonstrated by regarding the formula $G = \frac{\alpha}{C}$, with $C = \frac{I}{\frac{dY}{dt}}$ (Harrod's notation, but with α as savings-coefficient). The depression causes $\frac{dY}{dt}$ to fall relative to I, i.e. G decreases. Domar, however, thinks that I is kept up, which causes employment and therefore a higher $\frac{dY}{dt}$,

1 Harrod, Domar, p. 457.
2 Ibid., pp. 457, 458.

but not necessarily a $\frac{dY}{dt}$ high enough to let G increase. The junking process now in fact lowers I to $I.\frac{\sigma}{s}$, without affecting the beneficial effect of a higher $\frac{dY}{dt}$ too much. Thus, $\frac{I}{\frac{dY}{dt}}$ becomes smaller, and G is larger. A high rate of junking thus causes G to increase and is beneficial to stability.

The objection of Harrod can be put this way: If some of the new capital equipment is less or not more profitable than the old, then the old equipment will not necessarily be junked; and all the capital equipment may be somewhat under-employed. But this over-all excess capacity in an economy will have a much greater depressing effect on $\frac{dY}{dt}$, than that which merely a high rate of junking has on the future propensity to invest. Or again in the formula: $\frac{dY}{dt}$ becomes more smaller than does I, i.e. C increases and G decreases. At some rate of junking this second effect will become more significant than Domar's effect; somewhere then, there will be the optimum rate of junking which Domar did not recognize. By the same token, it is noted, Harrod did not (originally) see that there is a range where junking does inhibit the instability principle.

If the required rate of growth $\alpha\sigma$ cannot be achieved because of physical limitations ($\sigma < s$), Domar suggests the same remedies as Harrod did for bringing G_w in line with G_n. Kaldor seems to hold that "in the long run these will be brought together by a shift of income

distribution."[1]

The α in αs has more of an ex ante character than the α in $\alpha\sigma$. In αs, both α and s have to be reduced; thus lower the propensity to save (encourage spending) and develop those industries which require much capital and little labour per unit of output: the required marginal capital-output ratio must be increased by labour-saving, capital-using improvements. This leads to the conclusion that in the mature economies, with a low natural rate of growth, automation is not a cause for unemployment; it does cause the need for reschooling, but it does not cause unemployment.

Approaching the problem from the other direction, the maximum potential rate of growth $\alpha\sigma$ can be raised by speeding up the rate of technological progress, and the other factors, such as population growth, which determine the physical possibilities of growth. Otherwise the paradoxical situation will occur that because of a labour shortage, there is unemployment.

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[1] Harrod, Domar, p. 455.

CHAPTER III

SAMUELSON'S GROWTH MODEL;[1]

A COMPARISON WITH HARROD'S MODEL.

> "Mathematics is the steam shovel of logical argument; it may or it may not be profitable to use it." - R.G.D.Allen,
>
> Mathematical Economics

Samuelson's Model.

"The national income at time t, Y_t, can be written as the sum of three components: (1) governmental expenditure, g_t, (2) consumption expenditure, C_t, and (3) induced private investment, I_t."[2] Thus: $Y_t = g_t + C_t + I_t$. Autonomous investment is apparently neglected, although it is included in another discussion of this same model[3]. Consumption is taken as depending upon the income of the previous period: $C_t = \alpha Y_{t-1}$, where α stands for the marginal propensity to consume, and is assumed to be a constant. Investment depends upon the rate of change in

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1 P.A.Samuelson, "Interactions between the Multiplier Analysis and the Principle of Acceleration" in The Review of Economic Statistics, Vol XXI, No 2, May 1939, pp. 75- 78; reprinted in Readings in Business Cycle Theory (Blakiston Co, Toronto, 1944), pp. 261- 269 (page references are made to the 'Readings'); to be called 'Interactions'; and P.A.Samuelson, "A Synthesis of the Principle of Acceleration and the Multiplier" in The Journal of Political Economy, Vol. 47, 1939, pp. 786- 797; to be called 'Synthesis'.

2 Samuelson, Interactions, p. 265.

3 Samuelson, Synthesis, p. 791.

consumption as follows: $I_t = \beta (C_t - C_{t-1})$, where β, the consumption accelerator, is assumed to be constant. Direct dependence upon the rate of change in income (including both the rate of change in consumption and in investment) would have been closer to reality; this dependency is now achieved by substituting for consumption its relation to income. Since this relation assumes a lag, investment will now depend upon the rate of change in income with a lag: $I_t = \alpha\beta Y_{t-1} - \alpha\beta Y_{t-2}$. The national income will, neglecting g_t, be equal to:

$Y_t = \alpha(1 + \beta)Y_{t-1} - \alpha\beta Y_{t-2}$ [1]; $\alpha\beta = B$ is the income accelerator. The solution of this difference equation[2], which yields the path of Y over time, depends upon the roots (G_1 and G_2) of its characteristic equation and upon the two initial conditions: Y_o and $(Y_1 - Y_o)$. G_1 and G_2 in turn depend upon the coefficients α and β.[3] The initial conditions determine the values of the integration constants C_1 and C_2, to be introduced in the following paragraphs.

The different possible combinations of the values of α and β within the bounds of realism

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1 Samuelson, Interactions, p. 265.

2 Solutions are given of the equivalent differential equation. The distinction between difference and differential equations is not rigorously adhered to.

3 From the definition of G : $G = (Y_{t-1} - Y_{t-2}) / Y_{t-2}$, it is readily found that the general expression for G takes the form:

$G_{1,2} = \frac{1}{2}\left[-(2-\alpha-\alpha\beta) \pm \sqrt{(2-\alpha-\alpha\beta)^2 - 4(1-\alpha)}\right]$,

or shortened:

$G_{1,2} = \frac{1}{2}\left[p \pm \sqrt{-q}\right]$.

(i.e. $0<\alpha<1$, $\beta>0$) can be divided into four regions, each yielding a different qualitative behaviour of national income.[1] (See Figure 1).

FIG. 1

In region A ($p<0$, $q<0$), the solution is of the form $Y = C_1 e^{G_1 t} + C_2 e^{G_2 t}$, with G_1 and G_2 negative, different and real. With relatively small values of the (consumption) accelerator, there will be no cyclical behaviour; merely an asymptotic approach to the stationary equilibrium value of income. If there is a constant level of government expenditure or some other constant investment demand, the equilibrium will be $\frac{1}{1-\alpha}$ times the constant level of the expenditure. If there is additional expenditure followed by a complete

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1 Samuelson, Interactions, p. 268, and Samuelson, Synthesis, p. 792.

cessation, the income approaches the original equilibrium value of national income.

On the curve separating regions A and B ($p<o$, $q=o$), the solution, with two equal, negative and real G's is $Y = C_1 e^{Gt} + C_2 t e^{Gt}$.

In region B ($p<o$, $q>o$)[1], the solution is $Y = e^{pt}(C_1 \cos qt + C_2 \sin qt)$. The path of national income over time will be oscillatory and damped, eventually approaching the stationary equilibrium.

On the curve separating regions B and C ($p=o$, $q>o$)[2], the solution is $Y = C_1 \cos qt + C_2 \sin qt$.

In region C ($p>o$, $q>o$)[3], the solution is $Y = e^{pt}(C_1 \cos qt + C_2 \sin qt)$. The path of national income over time will be oscillatory and explosive; the cyclical oscillations take place around the original level of stationary equilibrium.

On the curve separating regions C and D ($p>o$, $q=o$), the solution $Y = C_1 e^{Gt} + C_2 t e^{Gt}$ contains two equal, positive, real G's.

In region D ($p>o$, $q<o$), the solution is of the form $Y = C_1 e^{G_1 t} + C_2 e^{G_2 t}$, with G_1 and G_2 positive, different and real. There are only explosive cumulative

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1 Here the two G's are complex.
2 Here the two G's are imaginary.
3 Here the two G's are complex.

movements either upward or downward; national income will grow cumulatively indefinitly at an ever increasing rate.[1]

The Influence of Lags.

Two features stand out in Samuelson's model, which makes use of a second order difference (or differential) equation with constant coefficients. In region C there are cycles. In region D, except in the special case where the initial conditions make one of the integration constants equal to zero, there will be an ever increasing positive growth rate (with a negative integration constant, Y will become ever more negative). Endogenously, and with constant coefficients, the second order Samuelson model can account theoretically for the occurrence of cycles and for Harrod's instability principle. These features could be accounted for in Harrod's model also, but only by changing the values of the coefficients in his first order equation, as a function of the growth rate, or of time. In Samuelson's model a stable case is also possible:

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1 The rate of growth $G = \frac{1}{Y}\frac{dY}{dt}$, is mathematically identical to a rate of compound interest (continuously compounded). Differentiation of the given solution readily shows that the 'effective' G is not constant. It increases from the smallest G (the minor rate of growth) to the largest (the dominant rate of growth). See also the section entitled 'A Closer Analysis of Samuelson's Model' and Figure 2.

in regions A and B the solutions are damped.[1]

In order to show that a cyclical behaviour is possible in region D, and that the explosive cycles in region C will, in reality, be limited, it is necessary to discuss the influence of non-constant values of α and β. It must be stressed however, that in the second order model this is only necessary to provide 'ceilings' (and 'floors'); in the first order model the instability principle itself had to be argued, using variable constants. Although Harrod may imply that lags are not essential in a dynamic model[2], the absence of lags did oblige him to resort to relatively complicated reasoning, thereby in essence changing the values of the constants s and v or C (these are the equivalents of Samuelson's $1 - \alpha$, and $\alpha\beta = B$).[3] It seems simpler to use an only slightly more complicated equation with a lag in the investment relation, as Samuelson proposed, thereby greatly lessening the need for changing the constants. Allen does not think that Harrod's theory is fully dynamic precisely because of the absence of lags; without lags the theory has no oscillatory feature and

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1 The solutions of first-order equations can only be cumulative and alternating; second-order equations also have the cyclical possibilities.

2 Harrod, <u>Towards</u>, p. 13; see also Chapter I, p. 4.

3 Harrod's C is determined by: $I_t = C(Y_t - Y_{t-1})$; Samuelson's B is determined by: $I_t = B(Y_{t-1} - Y_{t-2})$; B and C are thus not identical, although similar.

thus he considers lags essential to break the rigidity of the model.[1] Harrod however, certainly recognizes the existence and importance of lags: an example is found in the adjustment of the amount of capital stock to the need for it. This works as follows. If in a certain period people find that they have less capital, fixed or circulating, than they require in order most conveniently to execute turnover at its current level, they can do nothing in that particular period except to place extra orders or to step up their own production. Only if the people are on the line of warranted growth, will the ex post investment in a given period be equal to exactly the investment that is required in that period.

Before discussing the ceilings, it might be of interest to show that the same model, but without a second-order lag, does not have inherent cyclical solutions. Without a second-order lag the equations will be:

$$C_t = \alpha' Y_t$$

$$I_t = \beta'(C_t - C_{t-1}) = \alpha'\beta'(Y_t - Y_{t-1})$$

$$Y_t = C_t + I_t$$

Hence: $$Y_t = \frac{\alpha'\beta'}{\alpha'\beta' + \alpha' - 1} Y_{t-1}$$

Numerator and denominator are in all practical cases positive ($\alpha'\beta' = B' > 1$ and α' is positive). With $\alpha' \leqslant 1$, the coefficient $\dfrac{\alpha'\beta'}{\alpha'\beta' + \alpha' - 1}$ will always be greater

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[1] R.G.D. Allen, Mathematical Economics (Macmillan & Co Ltd, London, 1957), pp. 68,69,75.

than unity, and the path of Y over time will thus be a cumulative movement[1]; again if it is assumed that α' and β' are constants. Samuelson promises[2] to consider this possibility (the income-consumption relationship without a lag), since "for present purposes it (the time sequence involved in the relationship) must be squarely faced since entirely different results emerge from different assumptions"[3], but he does not do so. He merely states that "the time lag between income paid out and consumption is necessary if there is to be a cyclical pattern. Otherwise, there can be only a one-way movement away from equilibrium."[4] It has been stressed already several times that statements like this are only correct if it is added that the coefficients are assumed to be constants. This point seems to have been overlooked by both Samuelson and Harrod in their quoted statements. Omitting this point has contributed to cloud the arguments about the relative merits of models with and without lags.

Ceilings, Floors and the Stationary Equilibrium.

Variable coefficients α and β can provide

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1 A negative value of $\frac{\alpha'\beta'}{\alpha'\beta' + \alpha' - 1}$ will cause Y's to be alternatively positive and negative; this is a discontinuous effect, which does not resemble 'gradual' cycles.

2 Samuelson, Synthesis, p. 789.

3 Ibidem.

4 Ibid., pp. 796,797.

ceilings. The marginal propensity to consume (α) might increase as income decreases. A shift from region C to region D could be the result. Region D could be entered with initial conditions such, that an upturn will be caused. Although it is conceivable that α could then decrease with an increasing income, this does not seem too practical a consideration.

It is therefore more important to discuss the influence of investment through its effect on β. Since much of the reasoning will duplicate the discussion about the Harrod and Domar models, only a few remarks will be made about the effects of a non-constant consumption accelerator β. The value of β may change with changing levels of income. "Net investment can be negative only to the extent of deferred replacement or consumption. This provides a lower bound to the movements of our system."[1] Thus it would seem that Samuelson realizes that during the downturn the accelerator will stop functioning, since net induced investment cannot be negative by more than the numerical value of the depreciation. As income starts to rise, the accelerator will come into play again, once the excess capacity has been used up. However, Samuelson is not too clear about the role of the accelerator in regard to the lower turning point proper. He points out that Harrod

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1 Samuelson, Synthesis, p. 794.

does not use the acceleration principle in explaining the lower turning point; that he introduces the accumulation of deferred replacement as an extraneous factor for this explanation, and that because of this factor the recession must go to 'the bottom' every time; and that therefore the height of the boom has no effect on the depth of the depression. Samuelson does not agree with these conclusions of Harrod: "the height of the boom determines the rate at which income is falling at the point of stationary equilibrium, and this in turn determines the depths to which the system must fall before the upturn comes."[1]

This argument of Samuelson should not be taken too literally: in region D, the argument might be true, although there it does not of course follow from his model (where with constant α and β there are no cycles). On the other hand, in region C, it is not the height of the boom, but rather the degree of explosion and the length of time that α and β have been in play that determine the amplitude of the next oscillation. A more logical statement is that "the numerical relations of the acceleration principle and the multiplier generate the behaviour of our system."[2]

Another aspect of Samuelson's model is the level of stationary equilibrium. Although not including

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1 Samuelson, Synthesis, p. 796.
2 Ibid., p. 797.

autonomous investment in his multiplier—acceleration interaction analysis, Samuelson does regard the role and level of spontaneous investment as extremely important. The level of autonomous investment, that is the level of investment outlets, determines the average level of the system, which is entirely independent of the acceleration principle. Thus Samuelson regards the stationary equilibrium as a long-run trend determined by the secular level of spontaneous investment. This trend he considers to be far more important for the economy than the explosive action of the acceleration principle. Although Samuelson does not mention this, the determinants of investment outlets are population growth, supply of natural resources and technical progress: Harrod's determinants of the natural rate of growth G_n. Thus Samuelson realizes clearly that his analysis is only a cycle model; a pure trend theory, Hansen's model, will be discussed in Chapter IV.

A Closer Analysis of Samuelson's Model.

Solutions of Samuelson's second order differential equation are, in regions A and D, of the general form: $Y = C_1 e^{G_1 t} + C_2 e^{G_2 t}$; in region D, both G_1 and G_2 are positive. (See Figure 2).

Alexander[1] has called the smallest of the growth

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[1] Alexander, Accelerator.

rates (say G_1) the minor growth rate, the largest (G_2) the dominant growth rate.

> "The minor rate of growth is a critical dividing line between initial rates that will lead to an ever-increasing rate of growth approaching the dominant growth factor, and initial rates that will lead to declining rates of growth, and eventually to rapid decline."[1]

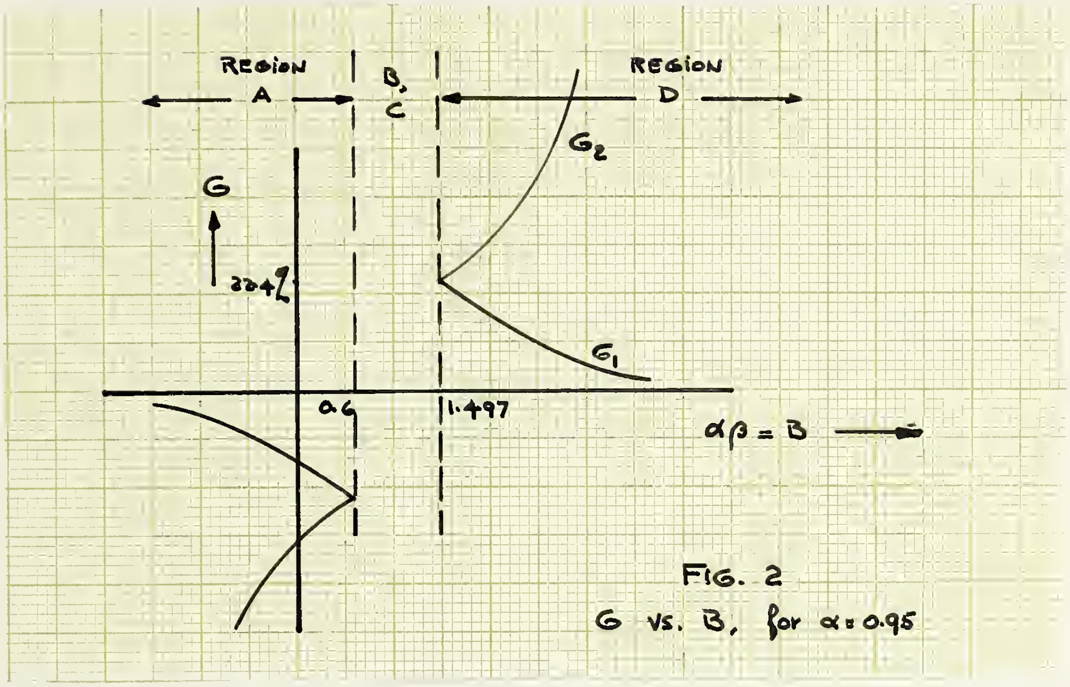

FIG. 2
G vs. B, for $\alpha = 0.95$

This characteristic fits Harrod's warranted rate of growth: only if G_w is precisely maintained will a

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1 Alexander, Accelerator, p. 179. It should be noted, that for initial rates of growth smaller than G_1 (the minor rate) the rate of growth will, contrarily to what is said inthis quote, also increase, approaching eventually also the (positive) dominant rate of growth. However, the integration constant C_2 will be negative, and income will, indeed, eventually decline rapidly, becoming more and more negative.

continuation of the same rate be induced. But if there is a small displacement from this warranted rate of growth, increasingly large displacements will be generated.

> "The frequently observed tendency for growth to be cumulative holds only for rates between minor and dominant rates. Within this range, the more investment there is the faster income will grow and the more investment will be required. Outside the range between minor and dominant rates, a given rate of growth or decline cannot sustain itself."[1]

Alexander apparently thought of a rate smaller than the minor rate (and not of a rate larger than the very large dominant rate) when he made his statement. He has implicitly assumed that investment cannot be indefinitely more negative, a phenomenon already discussed on p. 57.

It is Alexander's contention that earlier investigators of these relationships have been principally concerned with the minor rate of growth; that the dominant rate has been overlooked; and that this oversight is the consequence of the assumption that expenditure becomes income instantaneously (a zero income period). However, "the income period is an important characteristic of our economic system which must not be assumed away."[2]

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[1] Alexander, Accelerator, pp. 179- 180.
[2] Ibid., p. 180.

In his arguments, Alexander introduces the terms:

-income period:

"the time it takes that part of a given income which is spent on consumption to go through the economy and once again become income"[1];

-investment period:

"the time it takes for the investment expenditures induced by a given increase in national income to be made"[1].

Reconciliation of the use of these terms in Alexander's arguments with the solutions of Samuelson's equation seems impossible, since it would then be necessary to assume that with the income period Alexander means $\frac{1}{\alpha}$, and with the investment period $\frac{1}{B}$ ($B = \alpha\beta$); rather strange assumptions indeed. Compare: "If the income period is short, and the investment period very long, a steady approach to equilibrium is produced. For intermediate values of the time periods cyclical movements predominate."[1] This first conclusion is valid in region A, where α is large and B is small (see Fig.1 and Fig.2). However B, equivalent to Harrod's marginal capital-output ratio C, has the dimension time, and α is dimensionless; it would therefore seem to be an unhappy choice of words to call $\frac{1}{B}$ and $\frac{1}{\alpha}$ 'periods'. This

1 Alexander, Accelerator, p. 175.

explanation was devised, because the concepts income and investment period as defined above have no bearing on the solution of Samuelson's equation. The order of the equation is essential, the length of the period Δt is non-essential. In fact, in the limit when Δt approaches zero, the difference equation becomes the differential equation for which the regions as sketched in Fig. 1 and Fig. 2 are exact. The statement: "As a limiting case if investment depends on the (instantaneous) rate of growth of income, steady growth will be possible irrespective of the value of the accelerator, so long as it is positive"[1], must refer to Harrod's basic equation (postulate 2, on p. 5), which in this connection has the essential feature that only the first derivative of Y appears.

Overlooking Alexander's confusing terminology, it is an interesting point that:

> "For conditions that can yield steady growth, as the income or investment period is shortened, the dominant rate of growth increases without limit. So, as the income or investment period approaches zero the dominant rate becomes infinitly great. But if either period is assumed to be zero and completely ignored, the infinitly large dominant rate of growth is lost from sight. Only the minor rate of growth will then

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1 Alexander, Accelerator, p. 175.

be noticed. The recognition of the instability of the rate of growth thus discovered is an implicit reflection of the operation of the (hidden) infinitely large rate of growth."[1]

Referring to Fig. 2, and the solution $Y = C_1 e^{G_1 t} + C_2 e^{G_2 t}$ mentioned above, it is clear what is meant. Normally, when the growth rate is small (close to G_1), C_2 will be very small, so that the term $C_2 e^{G_2 t}$ is ignored. Since G_2 is very large, a small deviation of the true G from G_1 will 'soon' make the term $C_2 e^{G_2 t}$ 'dominant'. It pulls, as it were, the rate of growth off the value G_1.

The difference between Samuelson's model and the Harrod-Domar model can be summed up as follows.

Samuelson

> "pointed out that the interaction of the accelerator and the propensity to consume can lead to a variety of time patterns of the national income depending on the numerical values of these factors"[2],

while Harrod and Domar

> "investigated the rate of growth of income required to induce (or justify) enough investment expenditure to offset the savings that would be associated with the growing income, so that the growth of income could be maintained."[3]

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1 Alexander, Accelerator, p. 180
2 Ibid., p. 175.
3 Ibid., pp. 175- 176.

Alexander's conclusion is

> "that plausible values of accelerator and propensity to consume can indeed permit steady growth, but with constant prices (an assumption underlying this theory) they will lead to so rapid a rate of growth as to suggest that the assumed relationships cannot long persist."[1]

As a final remark about Samuelson's second order model, it can be said that exogenous 'shocks' are still necessary in regions A and B to keep the economy from being in stationary equilibrium[2], and that exogenous limits are necessary in regions C and D to keep it within bounds. The model is nevertheless more endogenous by far than is Harrod's or Domar's. A truly endogenous model would be very complicated; it would consist of differential equations with functionally determined, non-constant coefficients. Such an endogenous model would not give much economic insight, however. For gaining a better understanding of the economy Harrod's model is preferable.

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1 Alexander, Accelerator, pp. 175- 176.
2 Compare R.Frisch's 'erratic shocks' as a source of energy in maintaining oscillations; in R.Frisch, "Propagation Problems and Impulse Problems in Dynamic Economics" in Economic Essays in Honour of Gustav Cassel, 1933.

The Realization of Consumption- and/or Saving- and/or Investment Plans.

Harrod's warranted rate of growth is based on the equilibrium condition that both saving and investment plans are realized. The continued equation of saving and investment over time causes Harrod's growth model to become restricted to what is essentially a moving equilibrium[1]. Consumption expenditure is the factor that adjusts itself to the changing income: with $S_t = sY_{t-1}$ as ex ante and ex post saving, the ex ante consumption will be:

$$C_t = (1 - s)Y_{t-1} = Y_{t-1} - sY_{t-1},$$

the ex post consumption will be:

$$C_t = Y_t - sY_{t-1}.$$

When the actual rate of growth turns out to be different from the warranted rate, Harrod assumes that the saving plans will be realized. Thus the divergence between the desired saving and the desired investment must take the form of unintended investment, presumably in the form of an undesired accumulation or depletion of stocks or equipment. Since the investment plans are not realized, an adjustment must be made in the next period. This refinement was made by Baumol[2]. According

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1 Allen, Op. Cit., p. 78.
2 Baumol, Notes, pp. 506- 521.
W.J.Baumol, "Formalisation of Mr. Harrod's Model" in The Economic Journal, Vol. 59. 1949, pp. 625- 629; to be called 'Formalisation'.

to Baumol, Harrod assumed that the entrepreneurs plan to continue their output at the warranted rate of growth unless there is (positive or negative) excess demand for investment funds or savings: $I_t - S_t$. In this case they plan to depart from the warranted rate by an amount sufficiently great to make up for the excess demand, or the deficiency in investable funds:

$$Y_t = (1 + \frac{s}{C}) Y_{t-1} + I_{t-1} - S_{t-1}$$

With $S_{t-1} = sY_{t-2}$ and

$I_{t-1} = C(Y_{t-1} - Y_{t-2})$, it follows that

$$Y_t = \frac{C^2 + C + s}{C} Y_{t-1} - (C+s)Y_{t-2}$$ [1].

This second order equation gives a time path for income with, for likely values of C, the characteristics as specified by Harrod's analysis: namely, that the warranted rate of growth is self-perpetuating, once attained; and that if not attained, it will follow an explosive path upwards or downwards, depending upon the direction of the initial impulse.

Alexander[2] makes this same refinement. He calls the difference between justified (ex ante) and ex post investment in period t U_t: $U_t = C_r(Y_t - Y_{t-1}) - sY_{t-1}$.[3] It is, still assuming that savings plans are realized, equivalent to state that the rate of growth of income in period (t + 1) will equal the rate of growth in period t if investment ex ante equals saving ex ante in

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1 Baumol, Formalisation, p. 628.
2 Alexander, Model, pp. 724-739.
3 Ibid., p. 728.

period t. Then, redefining U_t as the difference between ex ante investment and ex ante saving, the 'gap' or instability theorem can now be worded thus: "if ex ante investment exceeds ex ante saving in period t, the rate of growth of income in period (t + 1) will be larger than that of period t, and vice versa."[1]

Alexander then introduces expenditure E_t. He gives the equation $U_t = E_t - Y_t$[2], with thus:

$$E_t = Y_t + U_t = Y_t + C_r(Y_t - Y_{t-1}) - sY_{t-1}.$$

The adjusted Y_t is then:

$$Y_t = (1 + \frac{s}{C_r})Y_{t-1} + U_{t-1} = (1 + \frac{s}{C_r})Y_{t-1} + E_{t-1} - Y_{t-1}.$$

This can be stated thus: when in period (t - 1) expenditure exceeds income, the entrepreneur will try to make up for this by raising his income in period t by precisely the amount of the deficit. This interpretation of the Baumol-Alexander refinement is attractive, in that it is simple. The impression is given by Alexander that 'of course' $U_t = E_t - Y_t$, and that therefore

$$E_t = Y_t + C_r(Y_t - Y_{t-1}) - sY_{t-1}$$[2];

but it seems more reasonable to regard this relation as the definition of E_t. Expenditure would then be defined as the sum of ex post consumption $(Y_t - sY_{t-1})$ and ex ante investment $C_r(Y_t - Y_{t-1})$. With this definition expenditure is not really 'out-of-pocket' expenditure; this would be ex post consumption plus ex post invest-

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[1]Alexander, Model, p. 730.
[2]Ibid., p. 731.

ment, and would thus be equal to income.

Expenditure here includes 'loss-of-investment': expenditure is the sum of ex post consumption plus ex post investment, plus (ex ante investment minus ex post investment). This last term is this 'loss-of-investment'. It resembles 'loss-of-income', although 'loss-of-income' could be assumed to be 'loss-of-investment' times the multiplier, instead of just 'loss-of-investment'. This problem can be solved by interpreting 'loss-of-investment' as depletion of stocks or equipment, which can indeed directly (without a multiplier) be regarded as expenditure. Expenditure in this sense need not be equal to income, and fits the refinement nicely. Harrod's income can be said to be determined by entrepreneurial decisions based on the behaviour of expenditure. And when expenditure equals income, the rate of growth is continued, i.e. it is a warranted rate of growth.

Samuelson follows Keynes in considering consumption and investment plans as the independent variables (thus realized) and saving and income as the dependent factors. The difference between desired saving and desired investment results in unintended saving by means of the variable income factor. Allen considers the case of adjustment through unintended saving far more realistic than the case of adjustment through unintended investment.[1]

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[1] Allen, Op. Cit., p. 79.

Alexander likes to generalize Samuelson's model by basing it "on the assumption that income of any period equals expenditure in that period"[1]: $Y_t = E_t$. And when income equals expenditure, then the rate of growth has been justified. By generalizing the Harrod and Samuelson model in this way, the fundamental construction of the two models is clearly shown and their basic similarity can be better understood, according to Alexander.

> "What the Samuelson model says will happen the Harrod model considers as a normal development from which deviations occur. If there is a steady (= constant) rate of growth that satisfies the Samuelson model it will be a warranted rate of growth in the Harrod model. If a given rate of growth is unstable in one, then it is unstable in the other. If the Samuelson model can be satisfied only by oscillating movements the same will in general be true of the Harrod model."[2]

Although Alexander says that he generalizes both models, he makes essential adjustments only to Harrod's model. That then the generalized Harrod model is similar to the Samuelson model is not surprising, since this really was the purpose of the adjustments.

However, Baumol sees a definite difference between the two theories: "despite some similarity in

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1 Alexander, Model, p. 732.
2 Ibid., p. 734.

premises, they cover essentially different grounds." Baumol regards Samuelson's attempt as very "ambitious in scope in that it attempts to describe what is in fact likely to occur", while Harrod's analysis is regarded "primarily as a normative study indicating the sort of conditions which must be satisfied by the course of the level of the national income through time in order for investment demand to be satisfied."[1] The major point of difference, according to Baumol, lies in the assumed consequences of a divergence between desired saving and desired investment. Summarizing this difference: Harrod assumes savings plans to be realized, Samuelson assumes investment plans to be realized.

Baumol then analyzes the more general case of how the effects of a difference between desired investment and desired saving are divided between undesired saving and undesired investment; in this case only the consumption plans will be realized. Baumol's refinement consists of two parts. First of all he uses a more sophisticated investment relation, including the dependence of investment on the level of income, and including autonomous investment; this by itself already gives a closer resemblance to reality. Secondly, the R_t idea is used: R_t is the same as the U_{t-1} introduced earlier. Baumol also uses lags as being inherent in the system, thereby following Samuelson's idea, although Baumol

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[1] Baumol, Notes, p. 516.

realizes that it is only essential "to approximate the situation at a point of time."[1] It must be pointed out that many more refinements are possible and that they will all help to bring the theoretical model closer to reality.

Baumol assumes that output (income) during any period is based on the producers' estimates of consumer and investment demand: $Y_t = C_{pt} + I_{pt}$. Consumer demand itself is based , as in Samuelson's analysis, on income in the previous period, and is therefore realized: $C_t = \alpha Y_{t-1}$; but the producers' estimates of what consumption demand will be are based on the consumption in the previous period: $C_{pt} = C_{t-1}$ and thus $C_{pt} = \alpha Y_{t-2}$. The producers' estimate of investment demand (it is noted that investment demand is already a producers' decision) consists of induced investment (dependent on the rate of change in income in the preceding period); of the investment dependent on the level of income in the previous period; of autonomous investment (autonomous at least in the short run); and of the factor R_t, which represents the allowance which is made for the net investment demand that was not satisfied (or too much so) in the preceding period. Thus:

$$I_{pt} = C(Y_{t-1} - Y_{t-2}) + kY_{t-1} + K + R_t \; .$$

$$Y_t = \alpha Y_{t-2} + C(Y_{t-1} - Y_{t-2}) + kY_{t-1} + K + R_t$$ [2] ;

this is Baumol's fundamental equation.

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[1] Baumol, Notes, p. 507.
[2] Ibid., p. 517.

Divergences between desired and actual saving and between desired and actual investment may arise because of exogenous causes; for example because of changes in consumer preferences. Divergences may also arise because of causes which are inherent in the operation of the system; essentially because anticipations, and thus decisions made, are based on the experience of the past, whereas the situation at the present time is of primary importance. Thus both unexpected saving and unexpected investment will be the result of the divergences (it is better to call this 'unexpected' rather than 'undesired'). Not all of the unexpexted saving is undesired saving, since as a result of the unexpected income, saving desires will change. Undesired investment may arise because producers base their investment demand on the investment or consumption goods demand in the preceding period and either or both of these demands may have changed. An increase in either of these demands will cause unsatisfied investment demand; a decrease in the demands will cause undesired investment. R_t is now "the sum of all the past investment demands which have been left unsatisfied, minus the sum of the past undesired investment"[1]; it is the accumulated deficiency of investment, which can be positive or negative.

Baumol proceeds to eliminate R_t in the funda-

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[1] Baumol, Notes, p. 518.

mental equation. R_t is the difference between the actual investment demand in the preceding period, I'_{t-1}, and the actual investment made in that period, I_{t-1}; i.e.

$$R_t = I'_{t-1} - I_{t-1} .$$

I'_t, the actual investment demand in period t is the demand as it follows from the growth in income, $C(Y_t - Y_{t-1})$, plus the demand based on the level of income, kY_t, plus the autonomous investment, K, plus the unsatisfied demand left over from the preceding period, R_t. Thus:

$$I'_t = C(Y_t - Y_{t-1}) + kY_t + K + R_t,$$

hence,

$$I'_{t-1} = C(Y_{t-1} - Y_{t-2}) + kY_{t-1} + K + R_{t-1} .$$

This, it should be repeated, is not the same as the producers' estimate of the investment demand, I_{pt}, which is based not on actuality (period t), but on the preceding period, and is:

$$I_{pt} = C(Y_{t-1} - Y_{t-2}) + kY_{t-1} + K + R_t.$$

For I_{t-1} can be substituted (from:

$$Y_t = C_t + I_t \quad , \quad C_t = \alpha Y_{t-1} \quad) :$$

$$I_{t-1} = Y_{t-1} - \alpha Y_{t-2}.$$

Hence:

$$R_t = C(Y_{t-1} - Y_{t-2}) + kY_{t-1} + K + \\ + R_{t-1} - Y_{t-1} + \alpha Y_{t-2}.$$

The fundamental equation can now be written as an expression for Y_{t-1}; this expression contains R_{t-1}. Substituting this Y_{t-1} in the above expression for R_t, eliminates R_{t-1}. The result is R_t expressed in Y's and constants. Substitution of R_t in the fundamental equation then yields

the final result:

$$Y_t = 2\alpha Y_{t-2} - \alpha Y_{t-3} + C(2Y_{t-1} - 3Y_{t-2} + Y_{t-3}) + $$
$$+ k(2Y_{t-1} - Y_{t-2}) + K$$ [1] .

This is a third-order linear difference equation with constant coefficients. The course of income over time will depend on the values of α, C [2], k and K; and many paths are possible. Empirical studies will have to find realistic values for the coefficients to see what the course of national income will be; at least for those periods, during which the assumption of constant coefficients is sufficiently valid.

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1 Baumol, Notes, p. 519.

2 C is the same as Harrod's marginal capital-output ratio, and is not equal to Samuelson's $\alpha\beta = B$ as Baumol suggests. This was explained in footnote #3, p. 54.

CHAPTER IV

TREND MODELS.

"Ik . . . was begonnen aan een grote studie over de invloed van de invloed van de invloed van de invloed van Slarieper op Waldo Bonk op Dwavels op de jongeren" - L.Huizinga, Olivier en Adriaan.

Hansen's Trend Model.

Alvin Hansen's theory of 'secular stagnation' or 'economic maturity', better described as a theory of 'increasing under-employment' or a theory of 'growing deflationary gap' is a trend theory. Hansen analyzed the variables influencing the tendency towards increasing under-employment, which is apparent in the advanced economies and which has been implied as part of the 'stagnation' taking place at some advanced stage of capitalist development by the Classicals, Marx, Schumpeter, and Harrod and Domar. Higgins' more precise interpretation of Hansen's theory guides the following discussion.[1]

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[1] B.Higgins, "The Theory of Increasing Under-Employment" in The Economic Journal, Vol. 60, 1950, pp. 255- 274, to be called 'Theory'; and
B.Higgins, Economic Development (W.W.Norton & Co. Inc., New York, 1959); to be called 'Development'.

The following symbols are used[1] (and again a free enterprise economy with the government playing a minor role is assumed):

O_p - (maximum) potential output; thus $\frac{1}{O_p}\frac{dO_p}{dt}$ is Harrod's G_n.

O_a - actual output; $\frac{1}{O_a}\frac{dO_a}{dt}$ must equal Harrod's G.

I,S - net investment and saving respectively.

$\bar{I},\bar{S}$ - net investment and saving at full employment.[2]

L - the size of the labour force (in man-weeks); it is assumed that no change in the ratio of the labour force to the total population takes place.

K - the supply of known natural resources.

Q - the capital stock.

T - the level of 'technique' (or productivity): "any percentage increase in'T' will raise total output by the same percentage, without any change in quantities of any factors of production employed."[3]

N - unemployment (in man-weeks).

X_n - the potential output (per man-week) of the unemployed; X_n is not necessarily equal to O_p / L, the unemployed labourer does not necessarily have average productivity.

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1 Higgins, Theory, p. 256.

2 The quantities $\bar{I}$ and $\bar{S}$ must be seen as ex ante concepts, they are thought to belong to full employment. They can both be realized only if they are equal. Then there will be full employment and O_p will be attained.

3 Higgins, Theory, p. 256.

D - the degree of monopoly in the economy as a whole; this must be seen as a weighted average of ratios of price (average revenue) minus marginal cost to price for all firms: $D(t) = (AR - MC) / MR$.

r - the rate of interest.

p - the time-preference; this factor lumps together all the factors that influence the willingness to save with given levels of income and given interest rates.

Hansen's basic equations are[1]:

$$O_p = T.\ f(L,K,Q,),$$

where f stands for the production function. It is noted that L,K,Q, and T represent available supplies, and not the amounts of labour, resources, capital equipment and technology actually used in production.

$$O_a = O_p - N.X_n - M(D),$$

where M stands for the function "relating the loss of production through misallocation of resources to the degree of monopoly."[2]

The relation for O_a is not entirely correct. According to Higgins' definitions, $N.X_n$ stands for the potential loss in output through unemployed labour only. The very important potential loss in output due to unemployed natural resources, capital stock and technology is missing. Therefore N and X_n will pertain to all the

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1 Higgins, Theory, p. 256.
2 Ibidem.

factors of production in this chapter.

Loss in output can be the result of a misallocation of production factors due to the monopoly elements in the economy; for this reason the term M(D) is introduced. In the definition of M, Higgins mentions only a misallocation of resources. However, it is probable that not only the 'natural resources' were meant, but also the 'capital stock', and it is even possible that labour and technology were included in this particular use of the word 'resources'. Resources in the sense of 'production factors' would be the more likely and correct interpretation, since misallocated labour and a non-optimum use of the technology available would definitely also be the result of the monopoly present in the economic structure. Since monopolies have the power to prevent developments, they can actually prevent new investment. The refinement of including the 'monopoly' factor in the growth model was made by Domar.[1] This monopoly factor was not taken into account in Hansen's original thesis and it will be neglected in the further discussion of this model.

Thus:

$$O_p - O_a = N.X_n, \text{ and } \frac{d}{dt}(O_p - O_a) = \frac{d}{dt}(N.X_n) \quad .$$

The gap between potential and actual output over time is thus seen to be a function of the unemployment of the production factors. Higgins now states that

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1 E.D.Domar, "Investment, Losses and Monopolies" in Income, Employment and Public Policy; Essays in honour of Alvin H. Hansen (W.W.Norton & Co. Inc., New York, 1948).

"the volume of unemployment will be the product of some 'leverage' coefficient, ℓ, and the gap between savings and investment at full employment levels of output or real income, $\bar{S} - \bar{I}$,"[1]

i.e.

$$N = \ell(\bar{S}(t) - \bar{I}(t));$$

the leverage coefficient is a complicated multiplier.

The 'gap between savings and investment at full employment levels of output' is very similar to Baumol's 'gap' refinement: the positive or negative excess demand for investment funds (or savings) is made up for by the entrepreneurs.[2]

Differentiation with respect to time gives:

$$\frac{d}{dt}(O_p - O_a) = \frac{dNX_n}{dt} = \left[X_n\frac{d\ell}{dt} + \ell\frac{dX_n}{dt}\right](\bar{S}-\bar{I}) + \ell X_n\left(\frac{d\bar{S}}{dt} - \frac{d\bar{I}}{dt}\right).$$

Higgins considers it probable that $\frac{d\ell}{dt}$ is small in the upshot and thus unlikely to influence significantly the effects of any inflationary or deflationary gap. X_n might be thought of as being proportional to T:

$$X_n = kT;$$

accordingly:

$$\frac{d}{dt}(O_p - O_a) = \frac{dNX_n}{dt} = \ell k\frac{dT}{dt}(\bar{S}-\bar{I}) + \ell kT\left(\frac{d\bar{S}}{dt} - \frac{d\bar{I}}{dt}\right).$$

In Hansen's theory, only the trend factors $\frac{d\bar{S}}{dt}$ and $\frac{d\bar{I}}{dt}$ are analyzed, which makes it a theory of increasing underemployment rather than a full growth theory.

Net saving at full employment is considered

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1 Higgins, Theory, p. 257.
2 Baumol, Formalisation, p. 628.

to be dependent upon the potential output, the time-preference and the rate of interest:

$$\bar{S} = \bar{S}(O_p, p, r) \quad .$$

This is a more refined relationship for the savings factor than either Harrod, Domar or Samuelson had, who considered savings to be dependent on income (or output) only. However, after some considerations, Higgins[1] comes to the conclusion that p and r are probably not of much influence in their effects upon savings and that they may be neglected; this leaves O_p as the main determinant of $\bar{S}$.

The investment relation is as follows:

$$\bar{I} = \lambda(O_p) - \psi(Q) + \varphi\left(\frac{dL}{dt}, \frac{dK}{dt}, \frac{dT}{dt}\right) \quad .^2$$

In words: net investment at full employment will increase when the potential level of output increases and it will decrease with an increase in the size of the existing capital stock. There will be autonomous net investment as a result of population growth, resource discovery and technological progress. Investment induced by the rate of change in output is here considered to be so small that it can be neglected.[3]

The relations expressing $\bar{S}$ and $\bar{I}$ can be

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1 Higgins, Theory, pp. 258,259.
2 Ibid., p. 260.
3 Critics of the acceleration principle (which was used by Harrod) prefer the above type of investment relation. There is some disagreement on this point, however. See for example:
R.S. Eckaus, "The Acceleration Principle Reconsidered" in The Quarterly Journal of Economics , Vol. 67, 1953.

substituted in the equation determining $\frac{d(O_p - O_a)}{dt}$.
It is not possible to draw conclusions from the resulting expression due to the appearance of the unknown functions λ, ψ and φ. The following simplifying assumptions are now made:

$\bar{S} = \lambda(O_p)$, which makes $\bar{I}$: $\bar{I} = \bar{S} - \psi + \varphi$.

The functions φ and ψ will be defined as follows:

$\varphi(\dot{L}, \dot{K}, \dot{T}) = \dot{L} + \dot{K} + \dot{T}$, [1]

$\psi(Q) = Q$. [2]

With these simplifications:

$$\frac{d(O_p - O_a)}{dt} = \frac{d\,NX_n}{dt} =$$
$$= \ell k \frac{dT}{dt}(Q - \dot{L} - \dot{K} - \dot{T}) + \ell k T \left(\frac{dQ}{dt} - \frac{d^2L}{dt^2} - \frac{d^2K}{dt^2} - \frac{d^2T}{dt^2}\right).$$

Higgins in his discussion limits himself to $\frac{dN}{dt}$, which, with the same simplifications is equal to:

$$\frac{dN}{dt} = \ell\left(\frac{dQ}{dt} - \frac{d^2L}{dt^2} - \frac{d^2K}{dt^2} - \frac{d^2T}{dt^2}\right).$$

When O_p and its determinants L, K, and T grow at a decreasing rate, $\frac{d^2L}{dt^2}$, $\frac{d^2K}{dt^2}$ and $\frac{d^2T}{dt^2}$ will be negative and $\dot{L}$, $\dot{K}$ and $\dot{T}$ will be positive. Q and $\frac{dQ}{dt}$ will be positive; ℓ is positive.

$\frac{dN}{dt} = \ell\left(\frac{dQ}{dt} - \frac{d^2L}{dt} - \frac{d^2K}{dt} - \frac{d^2T}{dt}\right)$ is then positive.

This means that N, unemployment, increases.

$N = \ell(Q - \dot{L} - \dot{K} - \dot{T})$ is positive or zero; the right-hand member in the equation is therefore positive:

$\frac{d(O_p - O_a)}{dt}$ is positive, or the gap between O_p and O_a

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1 The notation $\dot{L}$ is used to denote $\frac{dL}{dt}$, etc.

2 Although Higgins does not explicitly state that he also makes these assumptions, he does seem to do this in at least part of his article.

increases. This is true even if there is full employment: even for $N = 0$, $\frac{dN}{dt}$ is positive; there will begin a growing unemployment if O_p grows at a decreasing rate. This is illustrated in Figure 3, which represents Hansen's conclusion.[1]

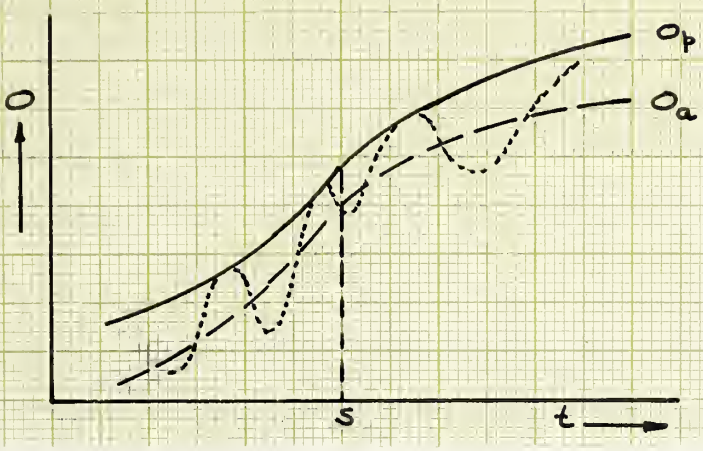

FIG. 3

Higgins puts this conclusion as follows:

"When population, supplies of new resources and technical knowledge are growing, but at a decreasing rate, savings will still rise; while at some point in time the unfavourable effects on investment of declining rates of growth and a growing stock of capital will tend to offset the favourable effects of a rising real income. From that point onward, net investment will tend to fall, and unemployment to increase. Thus an economy will tend to suffer increasing under-

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1 Higgins, Development, p. 177; see also: B. Higgins, "The Concept of Secular Stagnation" in The American Economic Review, Vol. 40, 1950, p. 164.

employment soon after the rate of growth falls off."[1] In Figure 3 O_p is the trend of potential income (i.e. the trend of national income at full employment and with constant prices); O_a is the trend of actual gross national income (at constant prices) around which economic fluctuations take place. What Hansen called 'stagnation' sets in at point s; yet neither the actual nor the potential trend shows actual stagnation at that point. Thus increasing under-employment can occur without actual stagnation of either the trend of potential gross national income or the trend of actual gross national income.

Although Higgins considers Harrod's theory an alternative formulation of Hansen's thesis in many respects, with both advantages and disadvantages in comparison, Higgins' main objection to Harrod's theory is that his equations "isolate no fundamental causal relations. In particular, they provide no explanation of investment decisions."[2] Harrod indeed does not analyze the determinants of long-run autonomous investment; however, all these factors together determine Harrod's natural rate of growth of output. Harrod does make the refinement to introduce a more realistic investment relation, by adding the influence of autonomous investment; but again, he does not consider its determinants. It is of interest that Hansen's autonomous

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1 Higgins, Theory, p. 264.
2 Ibid., p. 267.

investment $\dot{L} + \dot{K} + \dot{T}$ does not contain a constant, although a constant could of course be added; while what Harrod called autonomous investment was a constant only. Hansen's special contribution was his analysis of the determinants of the long-run autonomous investment function; a summary of this discussion is given in the following section.

Hansen's Long-run Determinants of Autonomous Investment.

The relationship between population growth and long-run investment[1] is the most original component in Hansen's long-run autonomous investment function, since before him population growth was usually a factor that did not enter the economic analysis. Population growth affects investment in two ways: it provides a growing labour force and it provides an increasing demand for goods and services. A shift from a rapidly growing population to a stationary or declining one may, instead of decreasing the demand for final products in general, alter the composition of the final flow of consumption goods in such a way that the ratio of capital to output as a whole may tend to decline (demand for personal services instead of demand for housing). However, it has been impossible so far to test any of these relationships emperically. But "for any country at any point of

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[1] Higgins, Development, pp. 183-188.

time there is some optimum rate of population growth: that rate of growth which is most conducive to the maintenance of a high level of employment and a rapid rate of economic progress (increase in output per capita)."[1]

Resource discovery[2] has generally been conceded to be stimilating to investment. Hansen however had more in mind than mere resource discovery. His argument was that "resource discovery in the special form of opening up a geographic frontier has particularly stimulating effects on private investment,"[3] since investment outlets are more difficult to find in a non-expanding economy. But the concept of a 'geographic frontier' is not easy to define. It may be seen as "an area within which there are increasing returns to both labour and capital with existing technical knowledge, population and tastes."[4] One of the conditions that might also have to be present is that it is an area "where the most advanced known techniques are not utilized."[4] The less tangible condition of, what Hansen calls 'frontier psychology' might also be necessary. This would mean in economic terms "a relatively low level of liquidity- and safety-preference,

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1 C.L. Barber, "Population Growth and the Demand for Capital" in The American Economic Review, Vol. 43, 1953, p. 136.

2 Higgins, Development, pp. 188-193.

3 Ibid., p. 188.

4 Ibid., p. 189.

or a relatively high marginal efficiency of capital, for any given set of objective conditions."[1] The geographic frontier can be contrasted with an economic frontier, "within which increasing returns would appear only with a change in techniques, population or tastes."[2] It should be noted that in developing his 'geographic frontier' concept, Hansen was especially interested in finding an explanation for the spectecular growth in the United States.

Technological progress may stimulate net investment, thereby offsetting savings, in three ways [3]:

- 1 by accelerating replacement.

- 2 by introducing new consumer goods so attractive that the propensity to consume is raised. However, a rising secular trend in the propensity to consume is unlikely as has been found by statistics[4] and as has been explained by Duesenberry[5]; but a short-run increase might very well take place. Very probably this will especially be the case in regard to consumer durables (at least in the mature economies). But if the innovations in the consumer goods take place at a constant rate, the consu-

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1 Higgins, Development, p. 190.
2 Ibid., p. 189.
3 Higgins, Theory, pp. 261- 263.
4 Simon Kuznets' data in Uses of National Income in Peace and War (Nat. Bureau of Economic Research, New York, 1942).
5 The demonstration effect: the relative position in a certain social group is maintained. See J.S.Duesenberry, Income, Saving and the Theory of Consumer Behavior (Harvard University Press, 1949).

mers may behave like entrepreneurs and replace the obsolete consumer durables by new ones with reserves built up for the purpose. Then there would be no rise in the consumption outlays as a result of the innovations.

- 3 by inspiring hopes of (temporary) monopoly profits for the first firms in the new field or those using the new technique; this is the so-called 'Schumpeter effect'. Since each innovation will absorb capital during its gestation period, it is not essential whether the invention, once in place, is capital using or capital saving; once in place only capital for replacement purposes will be absorbed. However, only if the stream of innovations is growing will there be a growing net investment (as far as the influence of T is concerned). A constant rate of technological progress means $\frac{d\dot{T}}{dt} = 0$ and so $\frac{\partial I}{\partial \dot{T}} \cdot \frac{d\dot{T}}{dt} = 0$ ($\dot{T}$ is written for $\frac{dT}{dt}$, this term represents the contribution of $\dot{T}$ to $\frac{dI}{dT}$). So no new net investment will be engendered as a result of a constant stream of innovations. A constant rate of technological progress will perpetuate past rates of growth (of investment), but it will not accelerate the rate of growth. Analogous arguments apply to a constant rate of population growth or a constant rate in the discovery of new resources. In a depression net investment could be zero; under a constant rate of technological progress it would thus (as far as the influence of T is concerned) remain zero. This is perhaps why 'secular stagnation' is most likely to make its presence felt after a period of prolonged depression.

Hicks' Contribution.

According to Hicks[1], Harrod's model should be seen as a supplement to the econometricians' theory (Samuelson). By emphasizing a few simple and central points, Harrod's theory is very attractive. For example, Harrod approached the business cycle as a problem of an expanding economy; thus the cycles should be seen as fluctuations about a rising trend. However, Harrod does not state what the actual path of the rate of growth of income will be in the centrifugal regions. It is Hicks' contribution to explore these regions more fully, whereby he introduces his 'ceiling' and 'floor' concepts. He also uses the long-range autonomous investment function as the determinant of the equilibrium output.

Hicks' model can be diagrammatically expressed as in Figure 4 [2]. AA represents the course of autonomous investment increasing at a constant rate. EE represents the equilibrium path of income or output and depends upon AA; it is deduced from AA by application of a 'super-multiplier'. FF represents the full employment (of all the factors of production) ceiling; this need not necessarily be above the equilibrium path EE. Hicks' cycle is based on the assumption of a value of the accelaration coefficient high enough to produce explosive

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1 J.R.Hicks, "Mr. Harrod's Dynamic Theory" in Economics, Vol. 16, May 1949; to be called 'Theory'; and J.R.Hicks, A Contribution to the Theory of the Trade Cycle (Clarendon Press, Oxford, 1950); to be called 'Contribution'.

2 Hicks, Contribution, p. 97.

behaviour (Samuelson's region C or D). An initial movement away from 'equilibrium' will thus result in an upward or downward path for the national income. Hicks' 'equilibrium' is merely an aggregative concept; it is his explicit contention that the economy is never in equilibrium.

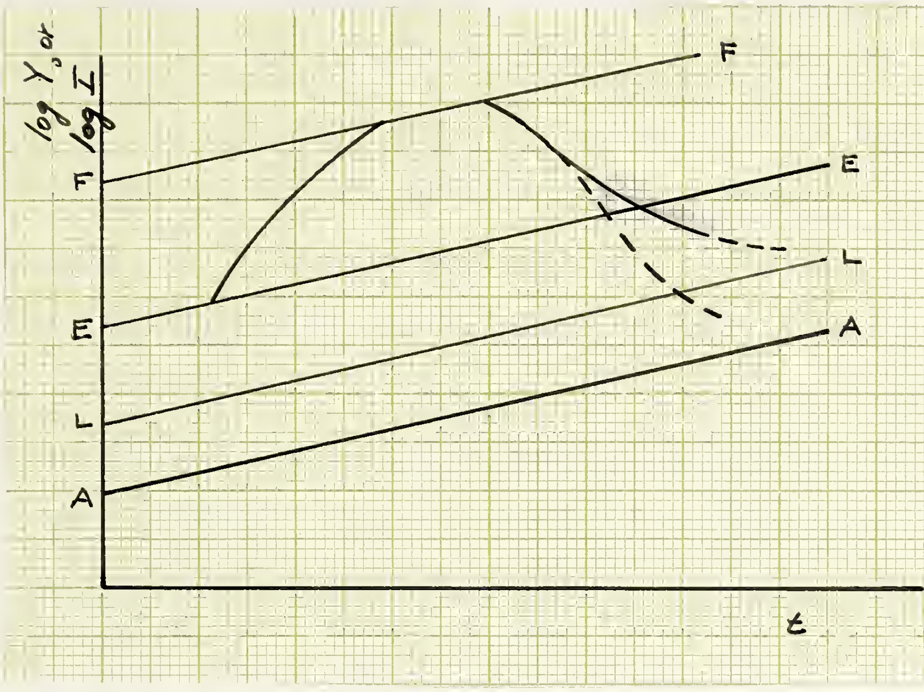

Fig 4

An upward expansion will eventually be restrained by a 'ceiling' in the form of a scarcity of employable factors of production. If the ceiling is high enough, the peak of the 'explosive cycle may (in Samuelson's region C) lie below it. In Hicks' diagram it is assumed that the ceiling rises at the same rate as the equilibrium output. Output will creep along the ceiling FF, but only for a limited time. Although supplemented by autonomous investment, the investment that is induced by the increase in output at this (ceiling) rate is insufficient

to support a growth of output along FF; the total investment is only enough to support an output which expands along the equilibrium path EE. The output will thus bounce off the ceiling back towards EE.

Once the output has started to fall, it will continue to do so until gross investment becomes zero; net induced investment is then negative and equal to the depreciation on the corresponding part of the capital stock. From then on, so long as the fall in output continues, induced investment ceases to depend on changes in output: the accelerator stops to function. The lower 'slump' equilibrium (LL) is formed by the usual multiplier, working on autonomous investment, minus the maximum disinvestment (depreciation), a constant. Especially if the decline in output has been rapid, it will take some time for the capital stock in excess of the requirements to wear out. But eventually the rising output along LL (rising, because the autonomous investment follows an upward trend) will bring the accelerator back into action: the recovery begins. With the accelerator at work the equilibrium path of output is EE and thus, subject to a positive induced investment, the system sheers away from LL.

In Hicks' model it is the cessation of the accelerator in the downswing which is the fundamental reason why the cycle repeats itself without damping (in region B) or explosion (in region C). Thus the ceiling proves to be not essential in region C. Due

to the suspension of the accelerator in the slump, each cycle begins with new initial conditions; each cycle will have a newly determined amplitude, so that no explosive behaviour will result.[1] In region D, however, the ceiling remains essential to keep the economy within bounds.

Schumpeter's Contribution.

It is especially Schumpeter's contribution[2] to regard 'innovations' through technological progress or resource discovery as the major determinant of autonomous investment. Continued growth will require a swelling stream of innovations and these will occur only, according to Schumpeter, with a growing supply of 'new men' to found 'new firms' and construct 'new plant and equipment'. Entrepreneurship is the vital force in the economy, since it is the entrepreneur who sees the opportunity for innovation, for applying the inventions and discoveries made, for introducing a new technique or a new commodity or an improved organization, or for the development of newly discovered resources. And according to Schumpeter, this supply of 'new men' will be choked off gradually in a mature capitalist economy

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1 See also J.S. Duesenberry, "Hicks on the Trade Cycle" in The Quarterly Journal of Economics, Vol. 64, 1950, pp. 464-476. Also Allen, Op. Cit., pp. 220,221.

2 Higgins, Development, Chapter V.

by a social climate antipathetic to enterprise. Such a 'hostile environment' will develop as a natural reaction to the entrepreneurs' very success. In this environment no innovations will be made and a more or less chronic depression will be the result.

The similarity of Harrod's entrepreneurial equilibrium G_w to Schumpeter's entrepreneurship as the vital force in a capitalist economy is remarkable. From Harrod's definitions it is clear that G_w depends entirely on the behaviour pattern of entrepreneurs; the production in any period is, up to the limitations imposed technologically, determined by the decisions of the entrepreneurs. G_w as a characteristic of the behaviour pattern of businessmen is determined by the propensity to save s: the characteristic of the behaviour pattern of savers; and by C_r: the technological characteristic of production. Harrod was apparently greatly influenced, when conceiving G_w, by Schumpeter, who considered entrepreneurship the vital force in the whole economy. If it is the entrepreneurial equilibrium G_w which, in relation to G and G_n, determines the state of the economy, then Harrod's growth theory is very close to Schumpeter's capitalist growth. According to Schumpeter[1], the supply of entrepreneurs depends upon the rate of profits and the 'social climate', and the 'social climate' can be considered to be reflected by

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[1] Higgins, <u>Development</u>, pp. 129-131.

the distribution of income. The ratio of profits to wages can be used as a convenient shorthand for all the factors influencing the social climate. Harrod's warranted rate of growth depends upon the productivity of investment (the reciprocal of C_r), which conveys the same idea as the marginal efficiency of capital. The marginal efficiency of capital in its turn is a determinant of the rate of profits and thus is also closely related to the ratio of profits to wages in the economy. And these are Schumpeter's two determinants of the supply of entrepreneurs. Alexander, who wonders about Harrod's G_w,[1] should thus not be too surprised. Conceding the importance of the entrepreneur as the stimulant of economic growth, Harrod's G_w seems quite acceptable.

Hicks' multiplier-accelerator model of the cycle is much improved upon, according to Fels[2], by including Schumpeter's innovation factor. In Hicks' theory innovation is one of the determinants of the autonomous investment curve AA, which in reality will have cycles of its own. In this treatment however, the interrelationships between the introduction of the innovations and the current business situation is ignored. Some of these introductions will be more, others less dependent on the current state of business. It is

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1 Alexander, Model, p. 725.
2 R. Fels, "The Theory of Business Cycles" in The Quarterly Journal of Economics, Vol. 66, 1952, pp. 25-42.

unlikely that business men react as strongly to a rise in demand (or a change in output) in all phases of the cycle as the theory of the accelerator would require, although the cumulative mechanism in the expansion phase is hard to reject. The accelerator is plausible only if it includes the factor of the innovation introductions. According to Schumpeter, the risk of introducing the innovations is at a minimum in the neighbourhood of the 'equilibrium' (Schumpeter's equilibrium however is Walrasian and thus not very realistic). The equilibrium will provide conditions which are especially favourable to the innovator: thus there will be a 'clustering' of innovations in the early stages of the upswing. Hicks' autonomous investment curve as a function of time alone is however quite justified, since the introduction of some innovations is indeed largely independent of current national income.

The upswing may be interrupted by one or more recessions. These can be caused either by short cycles of innovation introductions, or by a temporary decline in inventory accumulation as a result of the acceleration principle, or as a result of an unfavourable external event which changes price expectations for the worse. Such recessions do not develop into cumulative downswings because the investments associated with innovations will not be very much discouraged. Most innovations will continue to be introduced, at least

for a time, in spite of minor flurries in national income and they will provide a cushion for its fall. Once confidence is established, it is not easily shaken; the absence of monetary complications is of course also of importance.

Since at least a part of induced investment, including the important item of inventories, is geared to the rate of increase in output, the downturn may follow in accordance with the acceleration principle analysis. With 'innovations' at work in the cumulative expansion as another factor besides the accelerator, the full employment ceiling need not necessarily produce a downturn. If introduction of the innovations continues strong, the expansion phase may continue in spite of full employment. The innovations do not simply cause a rise of the ceiling; they cause an 'overshoot', which will stimulate the beginning of a recession. When the new factories are completed, investment will fall off; large volumes of consumer goods are thus turned out at a moment when they cannot be bought; prices will break and widespread disappointment with respect to profits will result; the recession sets in. The length of this 'overshoot', or 'innovation boom', is largely determined by the period of gestation, i.e. the length of time required for the new factories to be built. The boom driven by 'innovations' has thus a somewhat different turning point and mechanism than an

accelerator boom. In Hicks' model the end of a hump in autonomous investment may be seen as an exhaustion of investment opportunities, but these humps depend for their timing on the general business situation. The exhaustion of investment opportunities may be a sufficient condition for a downturn because intended saving then becomes greater than intended investment. The competing-down process in Schumpeter's theory also helps to cause the recession: the fact that costs increase more than prices in the upswing. This brings about a decline in profits which sets in motion a process of liquidation that may become cumulative (it is preferable now to think of (dis)equilibrium in the Walrasian instead of the aggregative sense). Thus full employment, the introduction of innovations, the competing-down process (and money stringency) form the four principal elements precipitating a major downturn.

Once set in motion, the downswing becomes cumulative on account of the accelerator. Minor upswings in the course of major downswings are unlikely; Hicks attributes this to a sudden rise in the liquidity preference. The innovations that are introduced are likely to be independent of the current business situation and may be included in the autonomous investment function. The course of the downswing and the ensuing upturn remain therefore the same as in Hicks' model.

Finally, it is of interest to note that autonomous investment can cause different sorts of cycles. It will, quite naturally, have cycles of its own. It can thus cause humps in the equilibrium path, even causing it to exceed the full employment ceiling. When national income is at the lower limit, a hump in autonomous investment can shorten the depression, or cause a short weak cycle which soon relapses to the lower limit. The extent and the timing of this hump can have many other consequences.

The Economy's Production Function.

The production function of an economy relates the total actual output (O, Y or P) to the size of the labour force (L), the supply of known resources (K), the capital stock (Q), and the level of technique (T). Thus O = f(L, K, Q, T). In this section L, K, Q and T (known as the 'productive factors' or the 'factor inputs') are quantities that are actually used in production; they form the actual inputs into the production process. This is, for L, Q and T, in contrast to the use of these symbols in the section about Hansen's growth model, where they stand for the supplies available.[1] It might be assumed that K already represented the

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[1] Higgins, Development, p. 169.

actually used (known) resources in Hansen's model. To elaborate a little, it is noteworthy that especially K and T are quantities that are difficult to define precisely. In fact T, being an abstract concept, is a quantity which is especially intangible, because for T one cannot easily speak of (for example) replacement value. However, T might be thought of in terms of productivity.

The labour force, the economy's known resources, and the capital stock produce together, combined in different proportions as determined by the existing level of technique, the economy's output. Consequently, L, K and Q are treated in a different manner than T: in addition to influencing O, T influences the functional relation between O, and L, K and Q. It is thus possible to write $O = g_i(T)f_i(L,K,Q)$, where the function g(T) could (possibly, but of course not necessarily) be a relatively discontinuous function. The possibilities of writing O as a function of L, K, Q and T are thus unlimited in scope.[1] Often the capital stock and the resources are taken together as one factor of production, namely as the factor 'capital stock'. In the following this simplification will be made and also the symbol 'K' will be used for this 'capital

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[1] In Hansen's model an explicit assumption (on p. 77) made it possible to write (n = 1): $g_i(T) = T$ and thus $O_p = T\ f(L,K,Q)$.

stock', complying with the notation in the literature referred to.

By assuming neutral technical progress Harrod uses in his growth model a constant marginal capital-output ratio and so does Domar, who uses a constant productivity ratio (a marginal output-capital ratio). A constant marginal capital-output ratio implicitly assumes that the increase in output is solely determined by the increase in capital stock (investment); this is of course not very realistic. Domar 'apologizes' for this simplification by stating that the effects of the changes in all the other factor inputs are reflected in the productivity ratio. Pilvin[1] has shown this diagrammatically.[2,3] (see Figure 5)

The slope of the line O'P represents the value of the marginal output-capital or productivity ratio.

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[1] H. Pilvin, "Full Capacity vs. Full Employment Growth" in The Quarterly Journal of Economics, Vol. 67, 1953, pp. 545-552.

[2] Ibid., p. 548; Pilvin depicts in his diagram a straight line OP; this makes the 'total' capital-output ratio a constant. However, only the marginal capital-output ratio need be constant; we prefer therefore to draw the straight line O'P, which does not pass through the origin. The indices in the diagram are clear from the next diagram (Figure 6); the first index corresponds to capital stock, the second to labour.

[3] Regrettably, Pilvin loosely speaks of 'capital', which can ambiguously be taken to mean investable funds or capital stock. He must have meant capital stock however. It might be remarked here that by the same token the term 'marginal capital-output ratio' as used in this thesis properly should be: 'marginal capital stock-output ratio'.

It can be seen that output must rise from the level P_{11} to P_{23} in order to utilize the new capital stock $K_2 - K_1$. Accompanying this change in output due to an

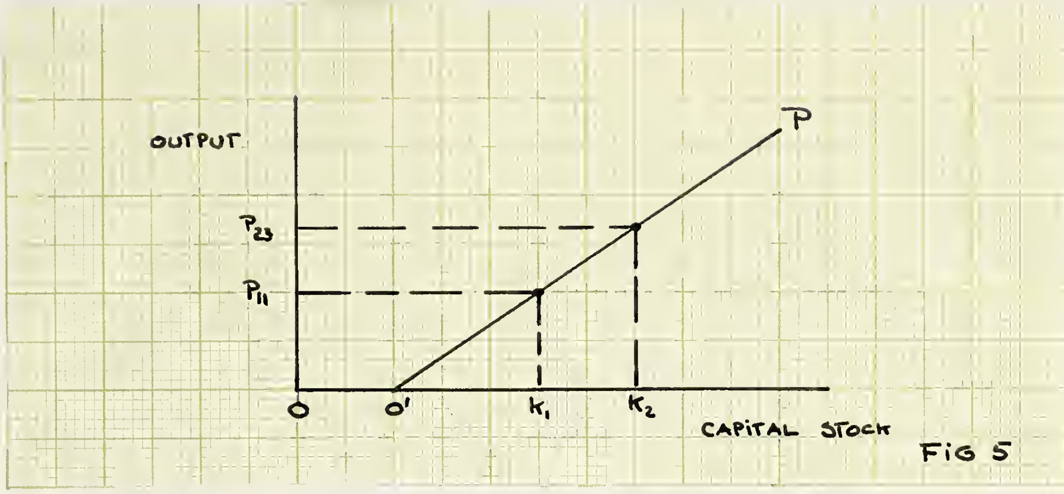

FIG 5

increase in the capital stock, there must have been one of the following changes in the labour supply and/or in technology:

<u>a</u> Without technological change, the production function can be represented by a fixed set of L-curves, where each curve shows the output possibilities of a fixed amount of labour with varying quantities of capital stock (see Figure 6)[1]. Originally the capital stock is K_1, the labour supply is L_1 and the output is P_{11}. With an additional capital stock of $K_2 - K_1$, the output will increase by $P_{23} - P_{11}$ only if the additional labour $L_3 - L_1$ is forthcoming.

<u>b</u> The same increase in output could also have been achieved with less additional labour, but with a

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1 Pilvin, <u>Op. Cit.</u>, p. 548.

concomitant productivity rise resulting from technological progress. Thus, if the labour force rose only to L_2, but technological progress resulted in a new L_2 curve intersecting O'P at R_{23}, the same increase in output would result.

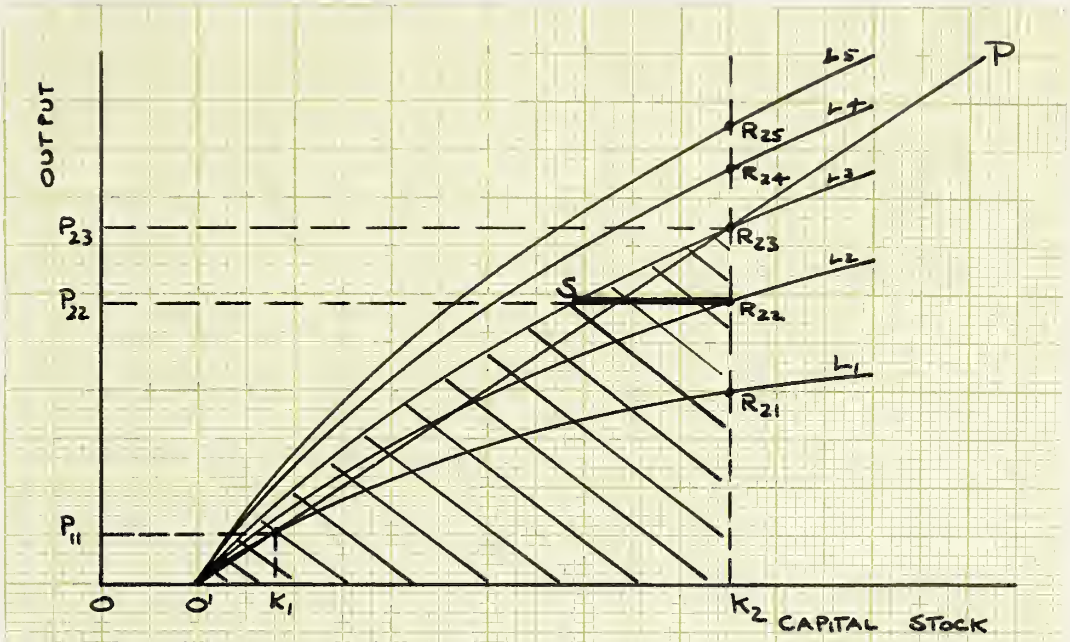

FIG. 6

c A third possibility of reaching the level P_{23} would be through technological change only; when the labour productivity increases by so much that the L_1 curve intersects O'P at R_{23}.

Joan Robinson[1] gives an interpretation of Harrod's concept of neutral technical progress. In her words, with a constant labour force:[2]"neutral progress in Harrod's conception results from an equal rate of

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1 J.Robinson, "Mr. Harrod's Dynamics" in The Economic Journal, Vol. 59, 1949, pp. 68- 85.
2 Ibid., p. 70.

increase in output per head at all stages of production."[1] She goes on to show that then: "the rate of capital accumulation required for the expansion of output made possible by the progress which is going on (with continuous full employment) is proportionate to the rate of increase in output," adding: "just as it is when population increases with constant technique."[2]

Referring to possibility a above, Pilvin states[3] that it is implicitly assumed in the Harrod-Domar model that the labour force has increased by $L_3 - L_1$ in order to bring about the increase in output; this seems a very arbitrary assumption. In the light of Joan Robinson's interpretation it would seem to be more logical to put this the other way around: it is implicitly assumed that when the labour force increases from L_1 to L_3, an amount of capital stock $K_2 - K_1$ will become available.

Regarding again the last diagram, it can be seen that when the assumption of a constant productivity ratio is dropped, the economy can operate in any point in the hatched area which should include the line OO', if the labour force has increased from L_1 to L_3, and an amount of capital stock $K_2 - K_1$ has become available. An output P_{23} would be attained if both labour and capital stock were fully employed. But if for example

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1 Robinson, Op. Cit., p. 70.
2 Ibid., p.71.
3 Pilvin, Op. Cit., p. 549.

an output P_{22}, smaller than P_{23}, is attained, the economy can operate anywhere on the line SR_{22}: in S labour is fully employed, capital stock is only partially employed; in R_{22} capital stock is fully employed and labour is only partially employed; in points between S and R_{22} both capital stock and labour are partially employed. It follows that under these circumstances P_{23} is the maximum possible output.

It is thus clear that Harrod has simply thought of Figure 5, diregarding labour altogether. This makes his definition of neutral technical progress[1] rather awkward, since it means a constant investment productivity, while the labour productivity can still change in any way (due to technical progress). Only in a model in which labour is neglected will the assumption of neutral technical progress have any value.

It is also Hamberg's complaint [2] that Harrod and Domar stress too much the capacity aspects of growth in their model[3]. Hamberg likes to make a clear distinction between the output growth rate for a full employment of a growing labour supply (E) and the output growth

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1 See also p. 15.

2 D.Hamberg, "Full Capacity vs. Full Employment Growth" in <u>The Quarterly Journal of Economics</u>, Vol. 66, 1952, pp. 444- 449.

3 Generally, with capacity is meant (fully employed) productive capacity of the capital stock, while Domar meant with 'productive capacity' full employment of all factors of production (see also p. 33).

rate for a full utilization of the capacity of the capital stock (U). E will be referred to as the full employment growth rate, U as the full capacity growth rate. Hamberg calls $\frac{E}{U} = \alpha$: the coefficient of the required rate of growth of income for a full employment of all the factors of production (here just labour and the capital stock). Assuming that the trend will be that the actual rate of growth will be as high as possible, the lowest of the two growth rates E and U will be the actual rate of growth; = 1 represents the ideal growth pattern from the secular point of view.

$\alpha = \frac{E}{U} < 1$ is similar to Harrod's $G_w > G_n$ case: the situation of secular stagnation. The full capacity growth rate cannot be reached, because it lies above the full employment growth ceiling: the culprit is a too slow growing labour supply. The actually realized growth rate falls short of the full capacity growth rate with the result that there is idle fixed capital and excessive stocks in the economy with all their well-known ill effects on investment. Keynes' 'spending' advice then follows logically, because too much is being saved relative to the economy's capacity to absorb these savings. Both Harrod and Domar center their discussion around the problem of a full utilization of the capital stock: how to continue to absorb the increments in the capital stock into production.

Harrod and Domar both recognize the other side of the picture: the case of $\alpha > 1$, which is similar to

Harrod's $G_n > G_w$ case; but they do not treat this case explicitly and Hamberg fills this gap. This case formed the classical enigma: secular expansion and inflation accompanied by a growing unemployment of labour; and this is the very present-day problem of the underdeveloped countries. Although the capital stock is continually used to its (growing) capacity, a secular growth of unemployed labour ensues: think of Marx's reserve army of labour. Because of the very large agrarian sector in almost all the underdeveloped countries, the increasing unemployment is for a large part disguised. It seems impossible to invest enough to yield a rate of growth of income sufficient to employ the rapidly increasing labour force. And especially because of the fact that the marginal consumption coefficient is practically one hundred percent, it is very difficult -however essential- to raise savings in order to increase the insufficient supply of capital stock. It is thus of necessity that in these underdeveloped countries the government has to play a major role in order to be able to collect at least some savings through taxation, forced and foreign savings. Naturally it is obvious that the relatively overpopulated, underdeveloped economies require capital-saving, labour-absorbing techniques for an optimum use of their resources. Not only should there be no conspicuous consumption, but also no conspicuous investment.

According to Pilvin, Hamberg's argument is

based on the implicit assumptions that there is no technological progress and that there are fixed production coefficients; that only on the basis of these assumptions equality between the growth of the labour force (man-hours) and of the capital stock will result in equality between the two required rates of growth of income; thus in $\alpha = 1$, i.e. only under these assumptions can α remain equal to 1. Pilvin's way of picturing Hamberg's argument is illustrated in Figure 7 below.[1]

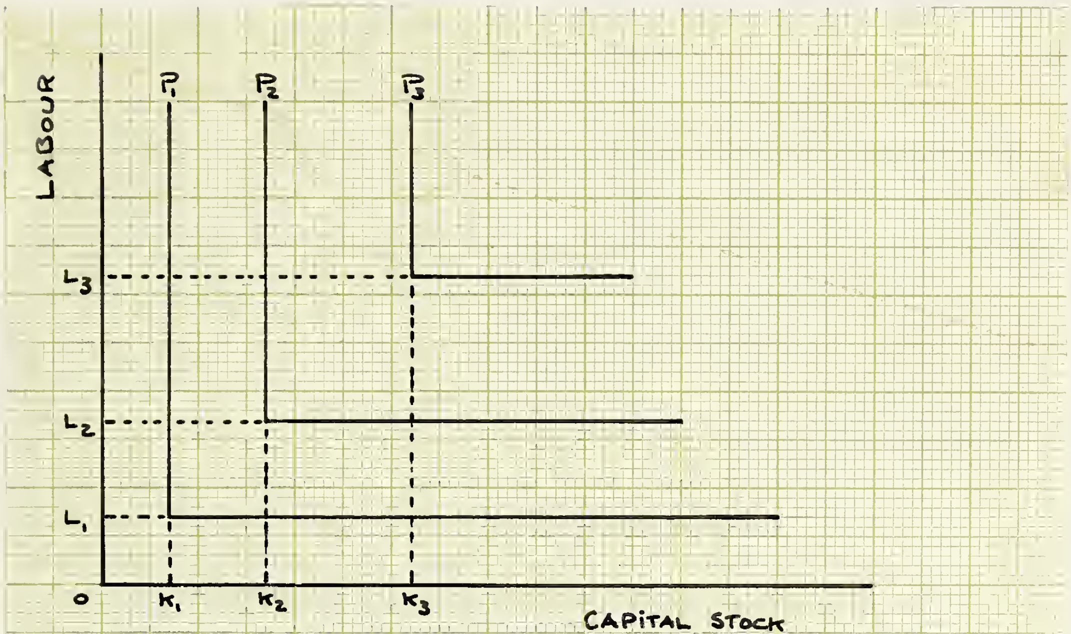

FIG. 7

A highly specific production function is required if unemployment of either labour or capital stock is to be avoided. Unless the factor inputs grow at the same rate, one factor, the one which grows at the faster rate, is partially unemployed.

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1 Pilvin, Op. Cit., p. 547.

Hamberg comments: "if it be argued that Pilvin is talking about movements along a given production function in a given state of technique, . . . , we may agree."[1] Hamberg maintains however, that he is not abstracting from technological progress, that indeed "advances in labor productivity are an important part of my argument, and when they are inthe picture, it distorts the argument to speak of movements along given production functions,"[2], since then there is a shifting production function. Growth in secular unemployment is the result of growth in the labour force and of advances in labour-saving technique. Labour-saving techniques imply changes in the capital-labour coefficients. A full employment growth rate in excess of a full capacity growth rate thus definitely is not based on the assumption of rigid coefficients. As a matter of fact the capital-labour coefficient would be falling in this case ($\alpha > 1$, the labour force increases more rapidly than capital stock); and vice versa for the case in which full capacity growth exceeds full employment growth.

According to Pilvin, the distinction between Hamberg's rates disappears if a 'normal' production function is assumed: one in which the production coefficients are perfectly flexible. The rates of growth

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1 D.Hamberg, "Full Capacity vs. Full Employment Growth: Some Further Remarks" in The Quarterly Journal of Economics, Vol. 68, 1954; p. 642.

2 Ibidem.

in the factor inputs need not be equal and it is unnecessary to assume that separate, and somehow independent, rates of required growth of income exist, one pertaining to each factor. "Whatever the individual rates of growth of factor supplies, a single rate of capacity increase results which is entirely consistent with full utilization of both factors."[1]

Pilvin assumes a production function[2]

$Y = f(L,K)$.

Thus:

$$\frac{dY}{dt} = \frac{\partial f}{\partial L}\frac{dL}{dt} + \frac{\partial f}{\partial K}\frac{dK}{dt} = f_L\frac{dL}{dt} + f_K\frac{dK}{dt}.$$

For $\frac{dK}{dt}$ can be substituted $I = \alpha Y$, with α: the savings-coefficient. Hence:

$$\frac{1}{Y}\frac{dY}{dt} = \frac{f_L L}{Y}\cdot\frac{1}{L}\frac{dL}{dt} + f_K\alpha.$$

In words:

> "the rate of income growth required to utilize both capital and labor fully and continually . . . is the rate of increase in the labor supply $(\frac{1}{L}\frac{dL}{dt})$ times the relative share of labor in the product, plus the marginal product of capital (stock) (f_K) times the propensity to save."[3]

Thus, by making use of a more general production function

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1 Pilvin, Op. Cit., p. 550.

2 Pilvin assumes the function $Y = f(L,K)$ to be linear homogeneous, which means that $\lambda Y = f(\lambda L, \lambda K)$. This makes f a function of the first degree in L and K (i.e. no products of L and K, 'within' or 'between', may occur). The consequence is that $\frac{\partial f}{\partial L}$ and $\frac{\partial f}{\partial K}$ are constants (do not depend on L and K). Economically: there are constant returns to scale. It does not seem necessary to make this restriction.

3 Pilvin, Op. Cit., p. 550.

than either Harrod, Domar or (according to Pilvin) Hamberg, Pilvin arrives at a 'required' rate of growth of income, which consists of a labour and a capital stock component. We will refer to this growth rate as G_p.

It is of interest to interpret Pilvin's expression in terms of the Harrod and Domar expressions. Analogous to Domar's σ:

$$\sigma = \frac{1}{I}\frac{dP}{dt} = \frac{\frac{dP}{dt}}{\frac{dK}{dt}} = \frac{dP}{dK} = \frac{dY_p}{dK},$$

in which both P and Y_p assume full employment of both labour and capital stock;

σ' can be introduced: $\sigma' = \frac{\partial Y_p}{\partial K} = f_K$.

Pilvin's rate of growth G_p equals by definition Harrod's G_n and Domar's $\alpha\sigma$; hence:

$$G_p = G_n = \alpha\sigma = \frac{f_L L}{Y_p} \cdot \frac{1}{L}\frac{dL}{dt} + \alpha\sigma'.$$

It is thus possible to write for Domar's σ:

$$\sigma = \frac{1}{\alpha}\frac{f_L L}{Y} \cdot \frac{1}{L}\frac{dL}{dt} + \sigma'.$$

While σ was the potential social average productivity of each invested dollar, in which a full employment use of the other factors of production was implied, σ' is the potential social average productivity of each invested dollar, in which the full employment use of labour is not implied, since labour is now treated as a seperate, explicit factor influencing the rate of growth of income. ($\sigma' = (\frac{\partial Y_p}{\partial K})_L$). Including technological progress, Pilvin simply adds T to his expression for O_p. It would seem that by writing Y = f(L,K,T) the term $\frac{1}{Y} \cdot f_T \cdot \frac{dT}{dt}$ should be added. Including this term would make σ' independent also of a full use of technological progress. ($\sigma' = (\frac{\partial Y_p}{\partial K})_{L,T}$).

Pilvin introduces an interesting graphical method, relating the production function of the economy to the savings-investment relationship (see Figure 8 below) [1]. The method can best be explained by some illustrative examples. Initially the labour force is L_1, the capital stock K_1 and the level of output P_1.

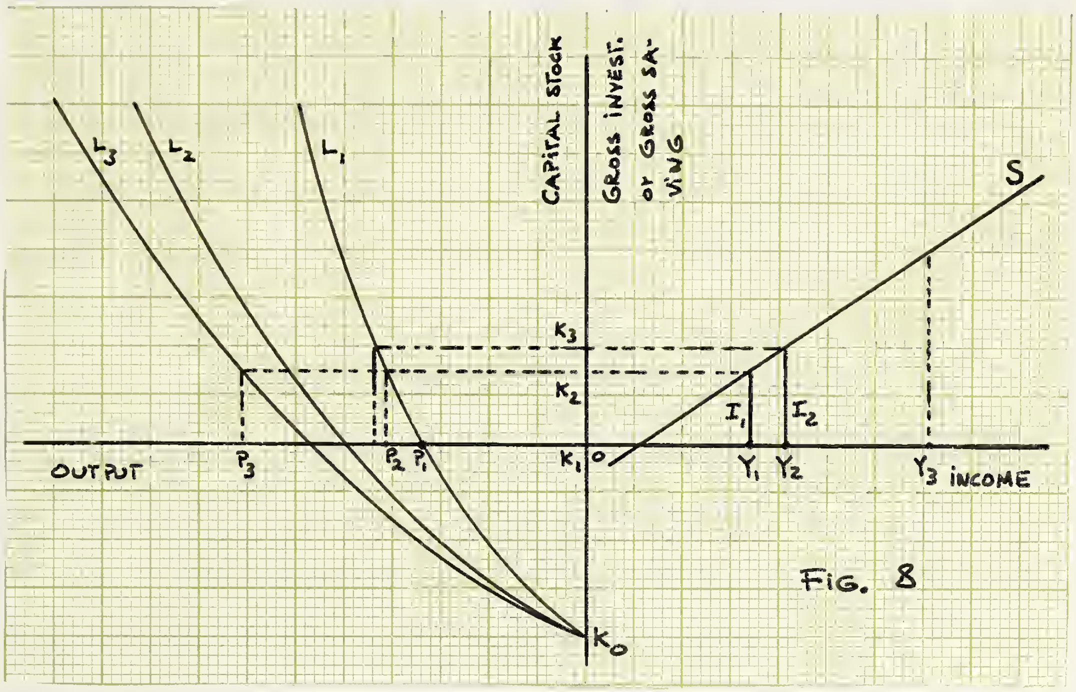

FIG. 8

Income equals output, or $Y_1 = P_1$. An investment of I_1 must be made in period 1 in order that Y_1 is attained; (at this point gross investment equals net investment). This raises the capital stock to K_2. The new level of capacity will depend on changes in the size of the labour force and on changes in technology. Assuming no technological change, the L-curves remain unaltered. If the labour force remains the same, there is a slight increase

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[1] Pilvin, Op. Cit., p. 551.

in output to P_2 and thus little required increase in income (and investment). I_2 must now be invested, the capital stock increases to K_3 and if the labour force in period 2 remains unchanged again, output will alter somewhat, depending on the shape of the L_1 curve. If the labour force increases when investment is I_1, for example to L_3, output will rise to P_3, requiring a large increase in investment to achieve the income level Y_3 corresponding to P_3. In the examples, full employment of labour and capital stock have been assumed. When using the diagram in this way, the growth rate of Y will, by definition, equal G_p.

Before saying more about Pilvin's 'model', it might be mentioned that Harrod[1] criticizes Pilvin's handling of the more general production function. Harrod apparently believes that Pilvin regards his growth rate as a 'steady growth rate'[2], as a growth rate which can be realized over time. Harrod then justly argues that the most important 'factor of production' is time; that a movement along a production function, which Pilvin's model implies, is impossible since it gives far too much importance to the role of the rate of interest (the price of time), even implies

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1 R.F. Harrod, "Comment" on "Full Capacity vs. Full Employment Growth" (by Hamberg and Pilvin) in The Quarterly Journal of Economics, Vol. 67, 1953, pp. 553-559.

2 Ibid., p. 554.

that it will be continuously falling over time. Only in one point of the production function will production methods be optimal. No technological progress is assumed to take place in this argument. Harrod's criticism seems to hold true: shifting upward along the L-curves in the diagram makes the production methods more capital-intensive. But it is not necessarily true: labour can grow such that the new L-curve to be used is intersected in a point where the production method is less capital-intensive. A similar argument holds for technological progress. It is difficult to prove exactly what Pilvin implied with his growth rate; his growth rate is so clearly just another expression for Harrod's G_n however, that it seems unlikely that Pilvin assumed that his growth rate would at all times be realized.

Pilvin's graphical presentation, when used for prediction and planning, is quite attractive and straightforward. Making the (admittedly far-reaching) assumption that planners can construct an 'employed labour productivity' curve and a gross savings curve, and that they can estimate the values of the unemployed capital stock and of the independent variable investment, the diagram might be used as a tool to calculate the actual growth rate and to analyze the economic problems. It would be possible to 'see' in the diagram which curves have to be reshaped in order to attain the desirable effects. However simplified this speculation may be, it is an entertaining expedition into the realm of practical application of the growth model.

CHAPTER V

SUMMARY

"..... the one unchangeable certainty is that nothing is certain or unchangeable" -

President Kennedy,
The State of the Union, 1962

In this thesis a study has been presented of the literature that followed in the wake of Harrod's attempt at a truly dynamic treatment of the economic system. The dynamics of an economy "in which the rates of output are changing"[1] requires special analytical tools. To explore this new economic frontier was Harrod's purpose.

Surprisingly, Harrod succeeded in presenting a growth model in a few bold basic concepts. Harrod's G, G_w and G_n concepts are a valuable addition to the arsenal of economic thought techniques; the feasibility of the different effects of their interrelationships provides an attractive toy; it becomes hard to resist translating other growth models into Harrod's concepts. As an example, the fears of both the Classicals and of Hansen in regard to population growth become readily understandible in the G, G_w and G_n terminology.

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[1] Harrod, Towards, p. 4.

This is not to say that Harrod's model does not have many shortcomings, mainly through not stating all the underlying assumptions and through a general lack of attention to detail. But luckily a stream of reviewers has tried to clarify and refine his model. Not all the reviewers' material has been exhausted, but it is felt that most of the really new additions to and clarifications of Harrod's theory have been covered. All these refinements help to round out Harrod's 'skeleton' model, but they do not necessarily improve the theory's applicability to the growth problems.

Although Harrod himself mentioned and shortly discussed his implicit assumptions of neutral technical progress and a constant rate of interest, it was not superflous that Joan Robinson and Pilvin both interpreted and explained the concept of neutral technical progress. Pilvin showed that the use of a constant capital-output ratio (the condition of neutral technical progress) implicitly assumed that labour and/or technology would have to adjust. Pilvin interprets graphically the implications of dropping the assumption of a constant ratio.

Harrod's hidden assumption in regard to the producers' plans and expectations was brought out by Alexander. Baumol discussed the possibility that a different assumption from Harrod's in this respect might provide a stabilizer of the instability principle. The possible stabilizing effect of a rising price level,

as interpreted by Alexander, is shortly discussed.

Autonomous investment, as a constant, is mentioned by Harrod. Baumol shortly shows the implications it might have on G_w. The determinants of long-range autonomous investment: population growth, resource discovery and technological progress, which are the same factors that determine Harrod's G_n, are by far best explained by Hansen. Schumpeter regards innovation by the priceless entrepreneur as its major determinant. Hicks also regards autonomous investment as essential for long-run growth; his treatment of this function however is vague, he prefers to regard it as a function of G_n.

Harrod's implicit assumption of realized saving plans with the resulting adjustment in investment plans is elaborated upon by Baumol and by Alexander. They suggest that the entrepreneurs will make up for the (positive or negative) excess demand for investment funds. Samuelson in his model, assumes that the investment plans are realized with the necessary adjustments made through income and savings. Baumol attempts a reconciliation of these two views: he sets up a model whereby the saving and investment plans are both only partially realized, thereby arriving at a model which resembles reality more closely.

Both Harrod and Domar were especially interested in the problem of 'secular stagnation' in the advanced economies. This problem is most clearly expressed in

Harrod's terminology by $G_w > G_n$. This case has the inherent depression characteristics of the high mass consumption economies. It also makes clear Hansen's fear of a declining population growth, which would lower G_n, while instead it should be raised.

The other case of $G_n > G_w$ causes the difficulties in the take-off and maturing economies. Too little investment keeps down the rate of growth with all its frustating effects. Hamberg feels that more attention should be given to this case. He introduces the full employment of labour and capital growth rates E and U, and treats the similar case of $\frac{E}{U} > 1$. It is interesting that Harrod's theory, although not intending to, did provide a framework also for interpreting these so very different growth problems.

Pilvin wanted to correct Harrod's (and Domar's) too one-sided treatment of 'capital' through the use of a more general production function. He arrives at a more explicit expression for Harrod's G_n, which consists of a labour and a capital component. An example of the use of a general production function, but then for different sectors of the economy, is Leontief's input-output analysis. Such a detailed analysis provides a fascinating insight into the essential flows in the economy.

Insight into the cyclical behaviour of the economic process can be gained by following the inter-

action between G and G_w. Harrod's instability principle forms an essential contribution. And although he did not especially pursue the actual rate of growth in the centrifugal regions, he did make some remarks to suggest in which way the turning points will be brought about. Short-run changes in the marginal savings coefficient, both ex ante and ex post, in their effects on G and G_w; and the decreasing mobility of the factors of production, are two important factors influencing the downturn, according to Harrod. Samuelson stresses the importance of the limit of negative net investment, with the result that the accelerator will stop functioning in the downturn. Hicks has been credited with especially exploring Harrod's centrifugal regions by introducing his 'ceiling' and 'floor' concepts.

The ceiling is formed by the eventual scarcity of the employable factors of production. Besides the fact that this ceiling does not seem really essential due to the irreversibility of the accelerator, Duesenberry[1] also doubts the actual existence of such a ceiling. Hicks thinks that bottlenecks will check investments before inflation sets in, but Duesenberry points out that there is no actual evidence to this effect. The problem may be to overcome a lag in production, but not to overcome an absolute shortage, because of the inherent

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1 Duesenberry, Op. Cit.

flexibility in the production methods. And although booms are accompanied by full labour employment, in these boom times low productivity labour has been drawn into higher uses. There may thus be a ceiling on the rate of increase in output, but there is not an effective ceiling on the level of output. Duesenberry points out that booms in reality have been brought to an end by a rise in the prices and the accompanying speculative forces; but not by the inability of investors to make physical investments. Schumpeter's 'innovation' factor also provides valuable insight into the character of the upswing and the resulting boom and downturn.

Hicks' floor (as does Samuelson's) consists of the zero gross investment level and the resulting suspension of the accelerator. The rising autonomous investment function will stop the slump eventually and induce an upturn. This rising trend in autonomous investment may be understood if one thinks of the irreversible consumption function, which keeps the income in a depression from falling back to the level of the previous depression.

In contrast to Harrod, who introduces his model out of a clear blue sky, Domar builds up his model of the interrelationships between 'stocks, flows and lags' far more logically, employing well-tried economic reasoning based on explicitly stated assump-

tions. He arrives at a growth mechanism very similar to Harrod's. Domar's derivation of a required rate of growth of income (and investment) to maintain full employment of all the factors of production is very instructive in providing insight into the different aspects of the economic structure. It is remarkably similar in approach to Pilvin's graphical tie-up of the economy's production function with the income-determining relationship. Domar's $\alpha\sigma$, which can be assumed to be equal to Harrod's G_n, is, if not explained as well as Hansen's long-run autonomous investment function, at least derived; while Harrod's G_n enters the picture only by definition. Thus Domar's $\alpha\sigma$ derivation can be seen as a more explicit statement of Harrod's G_n, although it must be said that Domar's σ concept is just as elusive as Harrod's G_n, and is also only introduced by definition (and tied up to Harrod's G_n by the definition of P).

But more important, Domar made no suggestion of any interrelationship pattern between r, αs and $\alpha\sigma$, except for his introduction of the 'junking process' concept: it provides a worthwile analysis of 'investment productivity' and its different possible effects on growth, and is Domar's special contribution. Thus, although all the ingredients were present for a cycle and trend interpretation similar to Harrod's G, G_w and G_n interrelationship, Domar himself did not work this out. In a way Domar started too close to the ground

to be aware of the flight his growth rate tools might take. Harrod on the other hand barely bothered with the ground construction, but saw his tools and started flying with them almost at once. His 'vision' is Harrod's major contribution.

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